M.A. /M.Sc. MATHEMATICS (PREVIOUS)
2010-2011

Note: There will be five papers in all and all papers are compulsory. Each paper will be assigned six hours per week.

PAPER-I
ADVANCED ABSTRACT ALGEBRA

TIME: 3 hours **Max. Marks: 100**

UNIT-I

External and Internal direct product of two and finite number of subgroups; Commutator subgroup ; Cauchy's theorem for finite abelian and non abelian groups, sylow's three theorem and their easy applications, Subnormal and Composition series, Zassenhaus lemma and Jordan Holder theorem.

UNIT-II

Solvable groups and their properties, Nilpotent groups, Fundamental theorem for finite abelian groups, Annihilators of subspace and its dimension in finite dimensional vector space, Invariant, Projection, adjoins, Singular and nonsingular linear transformation, quadratic forms and Diagenalization.

UNIT –III

Prime fields of characteristic zero and of prime number, Polynomial rings, Factorization theory in Integral domain, Prime and irreducible elements, Greatest common divisor and least common multiple, Euclidean domain, Principle ideal domain and Unique Factorization domain and their related theorems, Product of ideals and nilpotent ideals.

UNIT –IV

Definition and examples of Modules, sub module, Factor (Quotient) Module, Sub module generated by a set, Sum and direct sum of two sub modules, Homomorphism and isomorphism, Three isomorphism theorems in modules, simple, and cyclic and Finitely generated module, Fundamental theorem on finitely generated modules over Euclidean rings, Noetherian and artiman modules, Hilbert basis theorem.

UNIT –V

Field extension: finite and infinite, examples, Algebraic and transcendental extensions, Splitting field Separable and inseparable extensions, Normal Extensions, Perfect fields, Finite fields, primitive elements, Automorphisms, Galois theory of field extensions and its fundamental theorem, Solution of polynomial equations by radicals, Abel's theorem.

Books recommended:

1. Surjeet Singh and Quazi Zameeruddin : Modern Algebra
2. I.N.Herstein : Topics in algebra
3. R.S.Agrawal : Algebra
4. N. Jacobson : Basic Algebra Vol. I, II
5. S. Lang : Algebra IIIrd Edition
6. P.B. Bhattacharya S.K. Jain and Etc. : Basic Abstract Algebra (IInd Edition)

PAPER –II
REAL AND COMPLEX ANALYSIS

TIME: 3 hours **Max. Marks: 100**

UNIT-I

COMPLEX ANALYSIS : Complex numbers : The extended plane and its spherical representation, Analytical functions, Cauchy-Riemann equations, Power series including differentiation and integration within the circle of convergence, Conformal transformation, Linear, Bilinear, Exponential, Trigonometric and Joukowski's transformations. Riemann definition of integration, index of a point with respect to a closed curve and general form of Cauchy's integral formula.

UNIT-II

Simple connectivity, Cauchy's fundamental theorem, Cauchy's integral formula, Liouville's theorem; Morera's theorem, Taylor's theorem, Laurent's theorem; Poisson's integral formula, Maximum Modulus theorem, Rouche's theorem. Singularities, residues, Cauchy's theorem of residues and Evaluation of definite integrals.

UNIT – III

Metric spaces: Examples and properties of a metric space, Open sphere (ball or neighborhood) Open sets, closed sets and the related results, Continuous mappings Cauchy sequence and convergence, complete metric space, Compact spaces and compact sets, Baire's category theorem.

UNIT-1V

Measure Theory: Outer measure of a subset of R'Lebesgue outer measure of a subset of R ,Existence, non-negativity and monotonicity of Lebesgue outer measure, Relation between Lebesgue outer measure and length of an interval, Countable subadditivity of Lebesgue outer measure, translation invariance, Lebesgue outer measure-, (Lebesgue) measurable sets (Lebesgue) measure, Complement, union, intersection and difference of measurable sets, denumerable union and intersection of measurable sets, countable additivity of measure, the class of measurable sets as a algebra, the measure of the intersection of a decreasing sequence of measurable sets, some special classes of measurable sets, intervals, open sets, closed sets, Borel sets, F_6 and G_6 sets. Measurable functions; Different equivalent definition of a measurable function; Scalar multiple, sum,difference and product of measurable functions of measurable function. Measurability of a continuous function and measurability of a continuous image of measurable function.

UNIT-V

Supremum, infimum, limit superior, limit inferior and limit of a sequence of measurable functions. Convergence pointwise and convergence in measures of a sequence of measurable functions. Lebesgue Integral; Characteristic function of a set; simple function; Lebesgue integral of a simple function; Lebesgue integral of a bounded measurable function; Lebesgue integral and Riemann integral of a bounded function defined on a closed interval; Lebesgue integral of a non-negative function; Lebesgue integral of a measurable function; Properties of Lebesgue integral. Convergence Theorems and Lebesgue integral; the bounded convergence theorem; Fatou's Lemma: Monotone convergence theorem; Lebesgue convergence theorem.

Books Recommended:

1. George F-Simmons : Introduction to Topology and Modem Analysis, McGraw Hill Book Co.
2. S.I.Hu : Elements of Real Analysis
3. H.L.Royden : Real Analysis.
4. G.N.Purohit : Lebesgue Measure and Integration.
5. E.G.Phillips : Functions of a complex variable.
6. E.T.Copson : An introduction to the Theory of functions of a Complex variable.

PAPER –III

DIFFERENTIAL EQUATIONS

TIME: 3 hours **Max. Marks: 100**

UNIT – I

Partial differential equation: Existence and uniqueness of solutions, second order partial differential equations, boundary value problems, Green function and Cauchy problem.

UNIT – II

Calculus of variations: Linear functionals, Minimal functional theorem, General variation of a function, Euler – Lagrange's equation, Variational methods of boundary value problems in ordinary and partial differential equations. Variation problems in parametric forms.

UNIT –III

Series solutions of a second order liner differential equations near a singular, point (for benius method). Hyper geometric functions: Definitions of hyper geometric series and function; elementary properties of hyper geometric function; Integral formula for hyper geometric series, Linear transformations, contegeneous function relation, Linear relation between the solutions of hyper geometric differential equation Kumar's confluent hyper geometric function and its simple and basic properties

UNIT –IV

Legendre's polynomial Functions: Legendre's differential equation and associated Legendre's differential equations, Simple properties of Legendre's functions of first and second kind and the associated Legendre's function of integral order.

UNIT-V

Bessel functions, Generating function, Integral formulae, Recurrence relations; Addition formula and other properties of Bessel functions. Classical Orthogonal Polynomials, Generating functions and other properties, associated with the Jacobi, Laguerre, Legendre and Hermite Polynomials.

Books recommended:

1. Rainville, E.D. : Special Functions.
2. Sneddon, I.N. : Special Functions.
3. Sneddon, I.N. : Element of Practical differential equation.
4. Forsyth, A.R. : A Treatise of Differential equations
5. Gupta, A.S. : Calculus of variations with Applications
6. Bansal, J.L. : Differential equations Vol. II
7. Gelfand, I.M. and Fomin, S.V. : Calculus of variations.

PAPER – IV
GEOMETRY

TIME: 3 hours **Max. Marks: 100**

UNIT-I

The Axes of Plane Sections: Circular sections, Axes of central sections of a central conicoid, Axes of any section of a central conicoid, Axes of section of a paraboloid, Circular sections, Umblics.

UNIT-II

Generating Lines, The section of a surface by a tangent plane, Systems of generators of a central hyperboloid, Locus of the points of intersection of perpendicular, generators, The projection of generators, Generators' of the hyperbolic paraboloid.

Confocal Conicoids: The three confocals through a point,- Elliptic coordinates, confocal touching a given plane, confocal touching a given line, The parameter of the confocals through a point on a central conicoid, The normals. The self polar tetrahedron, The axes of an enveloping cone, The equation to the conicoid.

UNIT-III

Tensors; Transformation of coordinates, Contravariant and covariant vectors, second order tensors, Higher order tensors. Addition, subtraction and multiplication of tensors, Contraction, Quotient Law, symmetric and skew symmetric tensors: Conjugate symmetric tensors of the second order, Fundamental tensor, Associated tensors, Christoffel symbols, Transformation law of Christoffel symbols, Covariant differentiation of vectors and tensors.

UNIT-IV

Conoids; Equation to a conoid, surface in general, The degree of a surface, tangents and tangent planes, The inflexional tangents; the equations $\zeta=f(\xi,\eta)$. The indicatrix and representation by parameters. Curves in space, Equation to a curve, The tangent and its direction cosines, The normal plane-, contact of a curve and surface, Oscilating plane, Principal normal and binormal curvature, torsion, spherical indicatrices, frenet's formulae, signs of the curvature and torsion, formula for direction cosines of the principal normal and binormal , radius of torsion the relation $\sigma = + \eta \tan \alpha$ Circle of curvature, The osculating sphere and coordinates in terms of the arc.

UNIT-V

Envelopes: Envelopes of a system of surfaces with one parameter and its relation with characteristic, The edge of regression and its relation with characteristic, Envelope of a system of surfaces with two parameters and its relation with characteristic, skew and developable surface, Tangent plane to a ruled surface, Generators of developable surface, envelope of a plane with one parameter: criterion for $\zeta = f(\xi,\eta)$ to represent a developable surface and properties of a generator of a skew surface.

Curvature of surfaces, Curvature of normal sections through elliptic and hyperbolic points, Umblics, Curvature of an oblique section, radius of curvature of a given section through any point of a surface, Principal radii at a point of an ellipsoid: Lines of curvature of an ellipsoid, Lines of curvature on a developable surface, Normals to a surface at points of a line of a curvature, Lines of curvature on a surface of revolution, Principal radii and lines of curvature through a point of the surface,determination of umblics, Curvature at points of a generator of a skew surface, The measure of curvature at a point and expressions for the measure of curvature, Curvilinear coordinates, Linear element principal radii and lines of curvature.

Books recommended:

1. L. Robert, J-T.Bell : Coordinate Geometry of the three dimensions.
2. Bansal &Sharma : Differential Geometry.
3. B.Spain : Tensor Calculus.
4. J.L.Bansal : Tensor Analysis.

PAPER –V (A)

MECHANICS

TIME: 3 hours **Max. Marks: 100**

UNIT-I

Hydrodynamics: Lagrange's and Euler's, Methods; Acceleration, Equation of Continuity, Boundary surface, Stream lines, velocity potential. Euler's dynamical Equations, Bernoulli's Theorem, Lagrange's Equations under conservative forces, the motion once irrotational is always irrotational.

UNIT- II

Central Orbit, Kapler's Law of Planetary motion. Rigid Dynamics: Moments and products of inertia, Principal axes theorem, Parallel axes, Momental ellipsoid, D'Alembert's principle and the equation of motion.

UNIT-III

Motion in two dimensions under finite forces including sliding and rolling friction, Impulsive motion in two dimensions.

UNIT-IV

Principle of momentum and energy, Lagrange's equations in generalized coordinates.

UNIT-V

Michelson-Morley experiment, Lorentz-Fitgerald contraction, postulates of special theory of Relativity, Lorentz transformations, Mass - Energy formula, transformation formulas for momentum and energy. Minkowski's 4-dimensional continuum space, Space like and time like intervals, Relativistic Hamiltonian and Lagrangian.

Books Recommended:

1. S.L. Loney	:	Dynamics
2. A.S. Ramsay	:	Dynamics
3. A.S. Ramsay	:	A Text book of Hydrodynamics
4. M. Ray	:	Hydrodynamics
5. Gaur, Mathur & Goyal	:	Hydrodynamics
6. Bansal, Sharma & Goyal	:	Dynamics of a Rigid Body
7. Ray & Sharma	:	A Text Book of dynamics of a Rigid Body
8. M. Ray	:	Dynamics of a particle
9. Roy & Bali	:	Theory of Relativity

PAPER-V (B)

CONTINUUM MECHANICS

TIME: 3 hours **Max. Marks: 100**

UNIT-I

Cartesian tensors, index notation and transformation, law of Cartesian tensors, addition, subtraction and multiplication of Cartesian tensor, gradient of a scalar function, divergence of a vector function and curl of a vector function using the index notation. The identity, Stocks Gauss and Green's theorem. The continuum approach classification of continuous media. Body forces and surface forces, components of stress tensor, force and moment equation of equilibrium.

UNIT-II

The stress quadric, principal stresses and Principal axes, stress invariants and the stress deviator tensor, Maximum shearing stress. Lagrangian and Eulerian description of deformation of flow. The commoving derivative. Velocity and acceleration. The continuity equation.

UNIT-III

Strain tensors, the linear rotation tensor and rotation vector. Analysis of rotation displacement, Geometrical meaning of the components of the linear strain tensor, Principal axis theory for the linear strain tensor, Properties of linear strain tensors. The linear cubical dilation. Compatibility equations for the linear strain components. The rate of strain tensor and the vorticity tensor. The rate of rotation vector and vorticity. Properties of the rate of strain tensor.

UNIT-IV

Law of conservation of mass and Eulerain continuity equation. The momentum integral theorem and the equation of motion, Kinetic equation of state. The first and second laws of thermodynamics and the dissipation functions.
Application: (linear elasticity): Assumptions and basic, equations, generalising Hook's law for an isotropic Homogeneous solid. capatibility equations. classification of types of problems in linear elasticity. The principle of super position.

UNIT-V

The strain energy function, the uniqueness theorem P-L relationship and the work kinetic energy equation. Irrotational flow and the velocity potential, Kinetic equations of state and the first law of thermodynamics.
The equation of continuity. The equation of motion, vorticity-stream surfaces for inviscid flow, Bernoullis equations, Irrotational flow and the velocity potential, similarity parameters and fluid flow.

Books for References:

1. Q. Fredenic & T.S. Chang : Continuum mechanics: Allyn and Bacon, Inc, Boston.
2. Mase G.E. : Continuum Mechanics (Schaum series).
3. Sommer Field A : Mechanics of Deformable bodies.
4. Mortone E. Gurtin : An Introduction to Continuum Mechanics (Academic Press).