## **Differential Equations**

## UNIT- II

Ву

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Mathod of Shange of Independent Variables Consider the second order LDE.  $\frac{d\hat{y} + fdy + Qy = R - - 0}{dx^2}$   $\frac{dx^2}{dx}$ where f, R, R are functions  $\frac{d\hat{y} + fdy}{dx} = \frac{f(x)}{dx}$   $\frac{d\hat{y} + fdy}{dx} = \frac{f(x)}{dx}$   $\frac{d\hat{y} + fdy}{dx} + \frac{f(y)}{dx} = \frac{f(x)}{dx}$   $\frac{f(x)}{dx} = \frac{f(x)}{dx}$ (dz) 2 dy + dy . dz + P: dy dz + Ry

dx) dz 2 dz dz dz dz dy + Pidy + Dry = Ry -- 5)

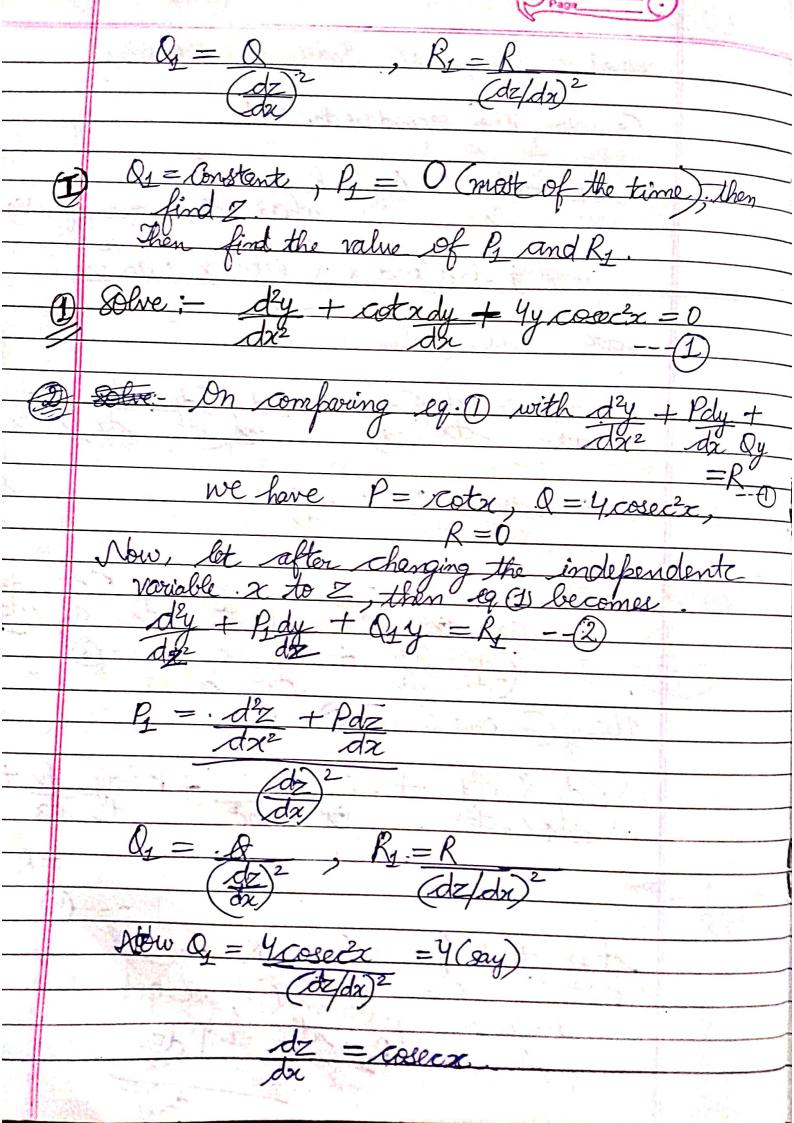
d22

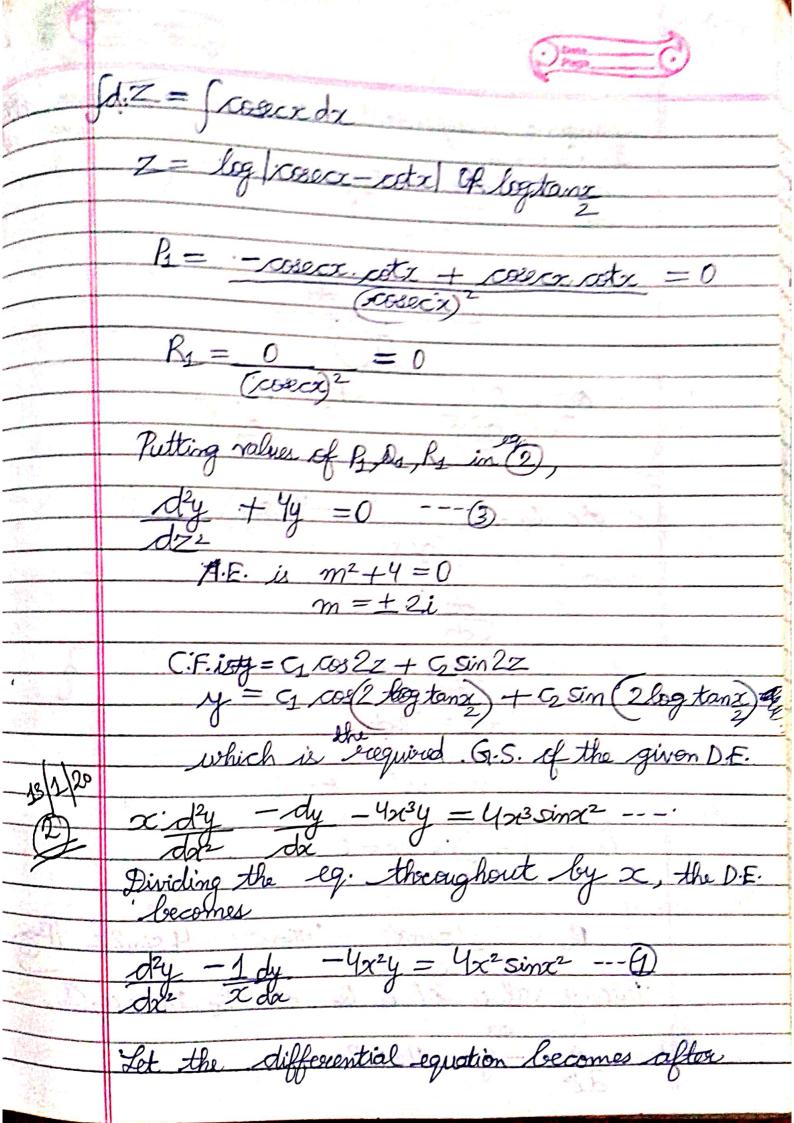
Comparing eq. (y) and (3), we get

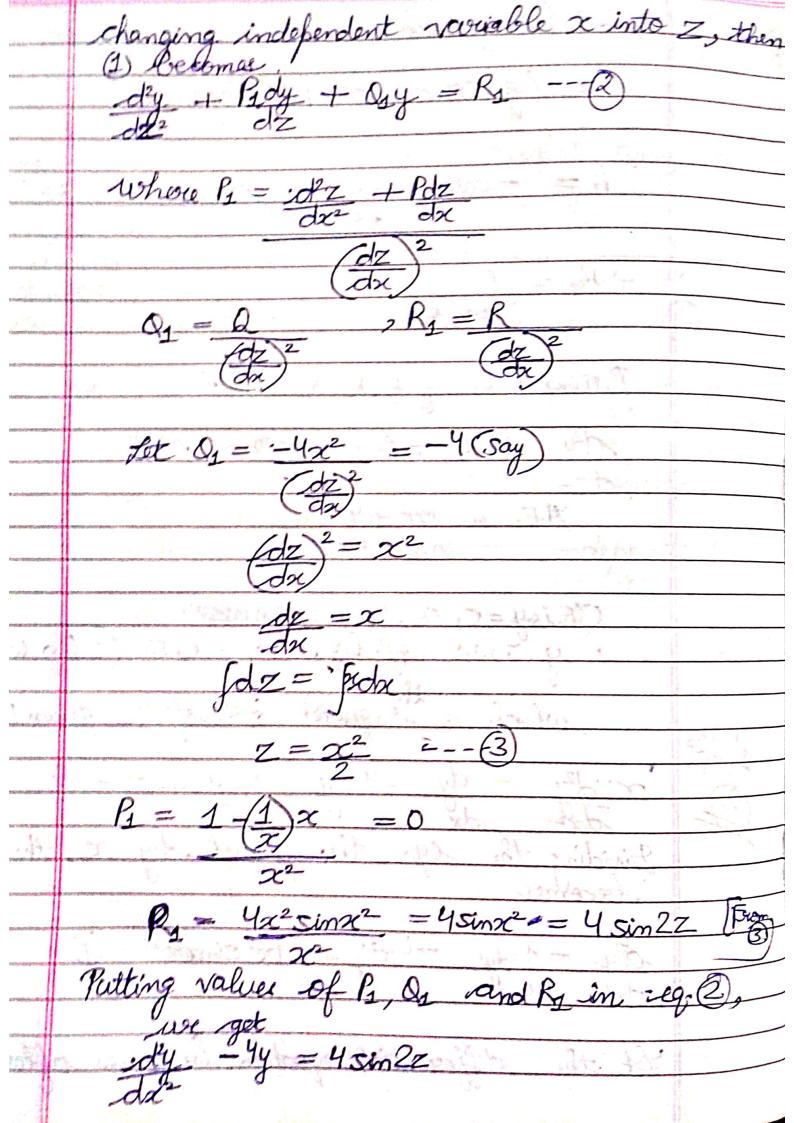
Pr = d2 + iP de

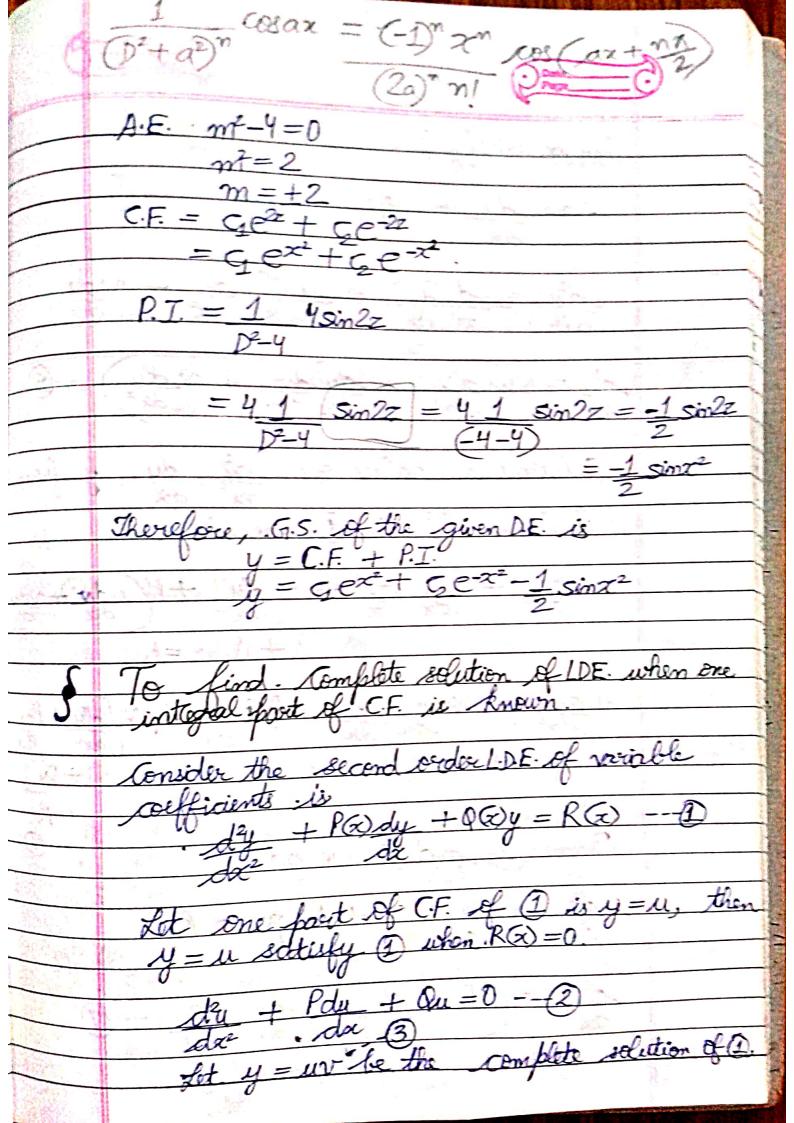
dx

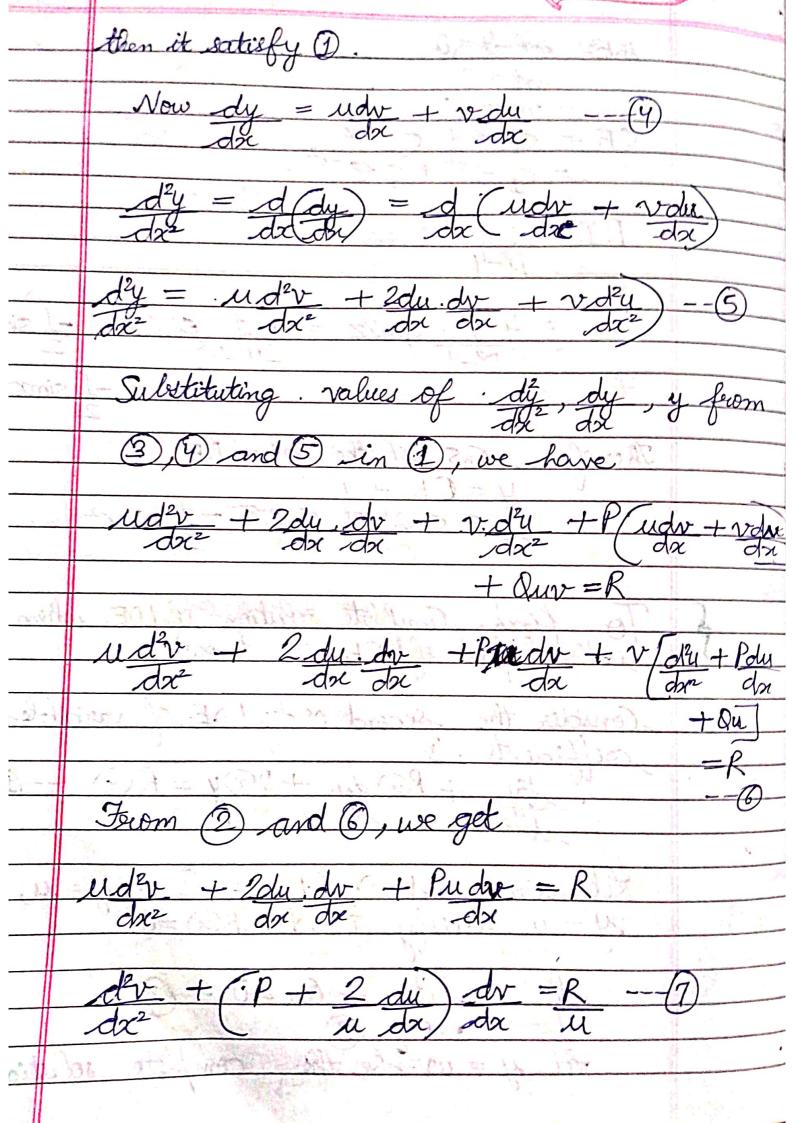
(dxx)

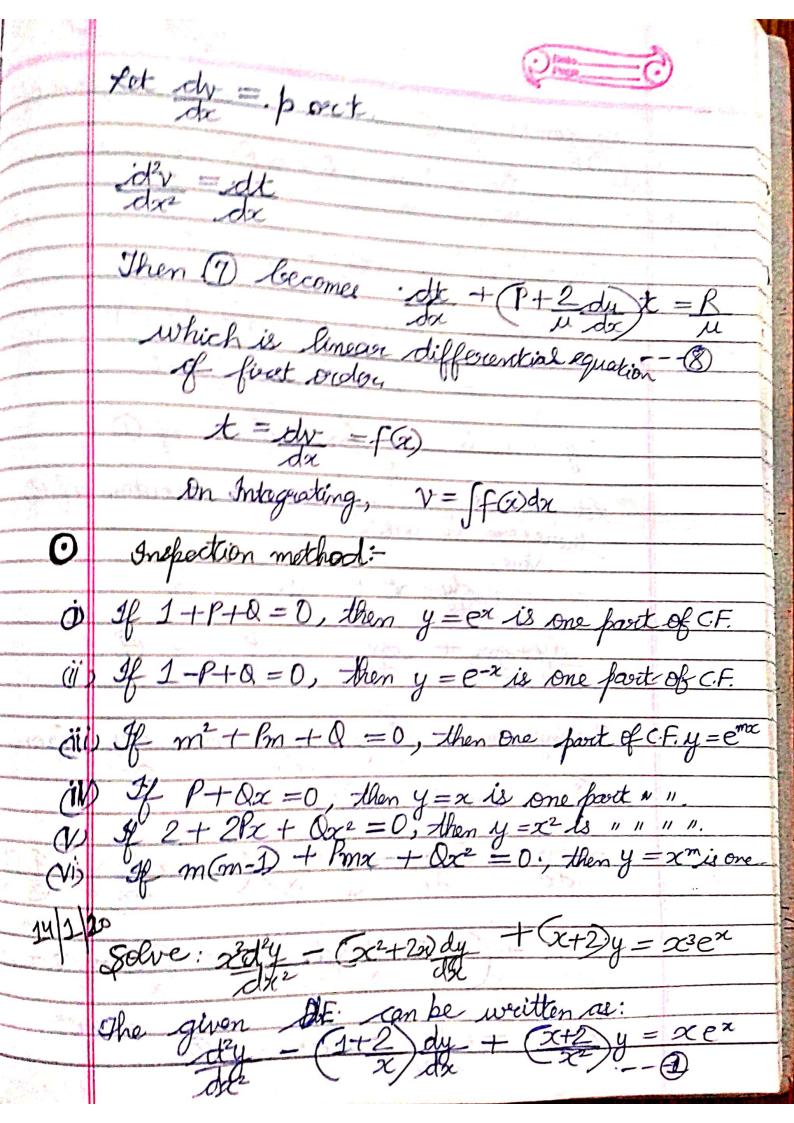


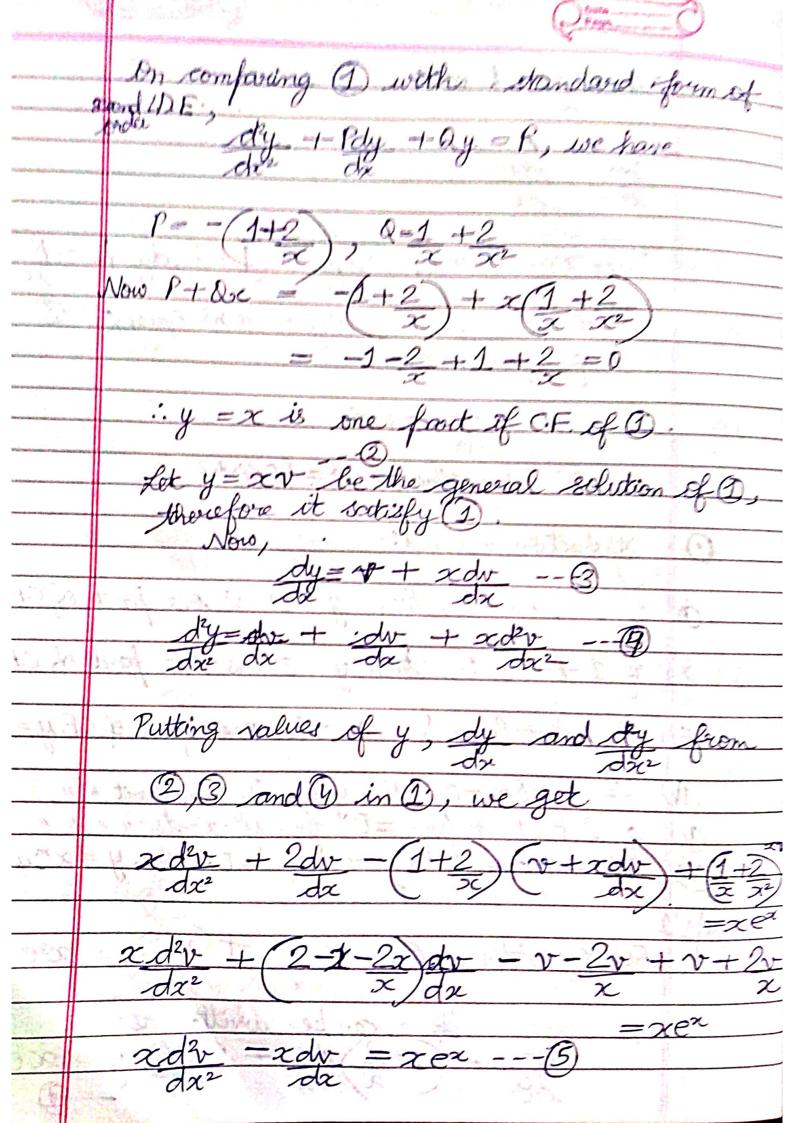


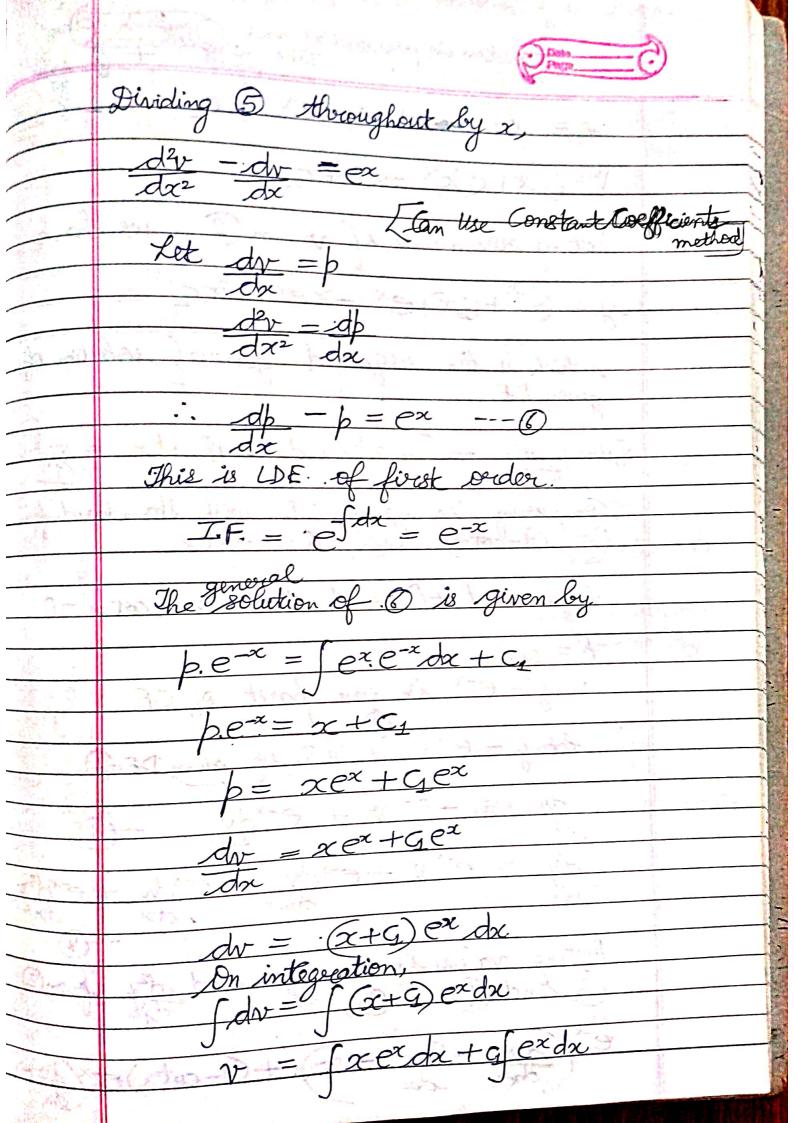




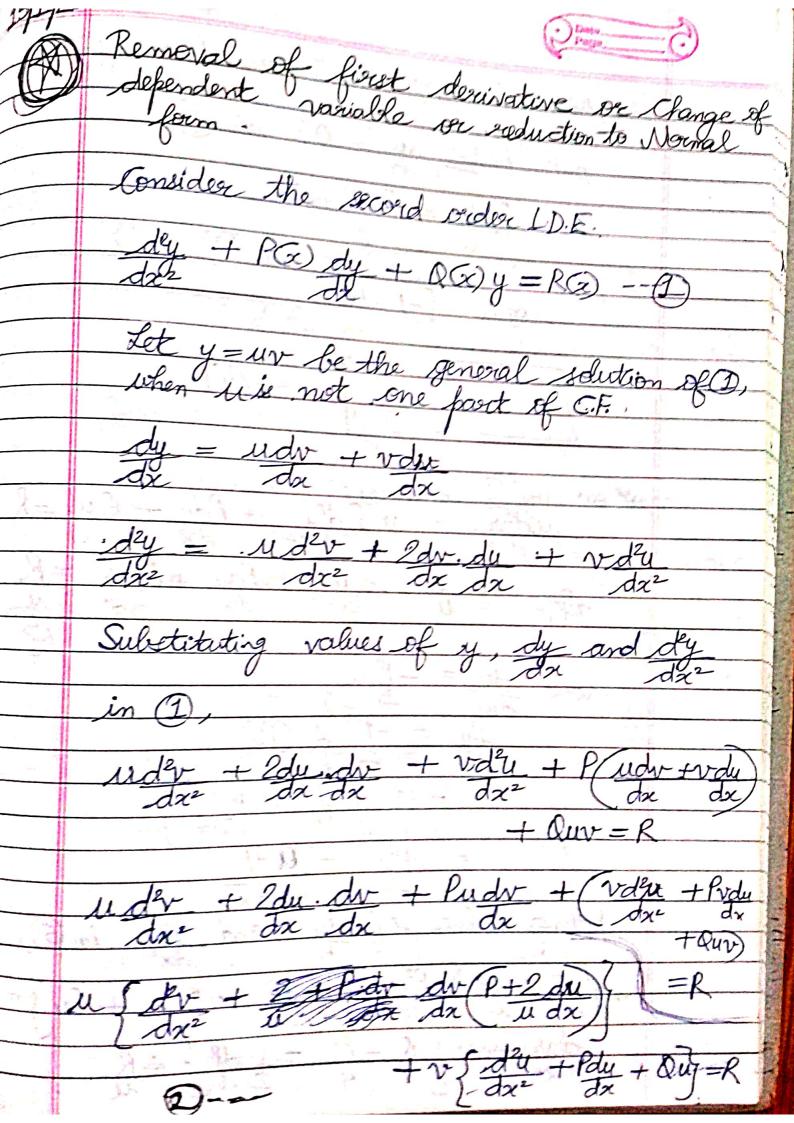


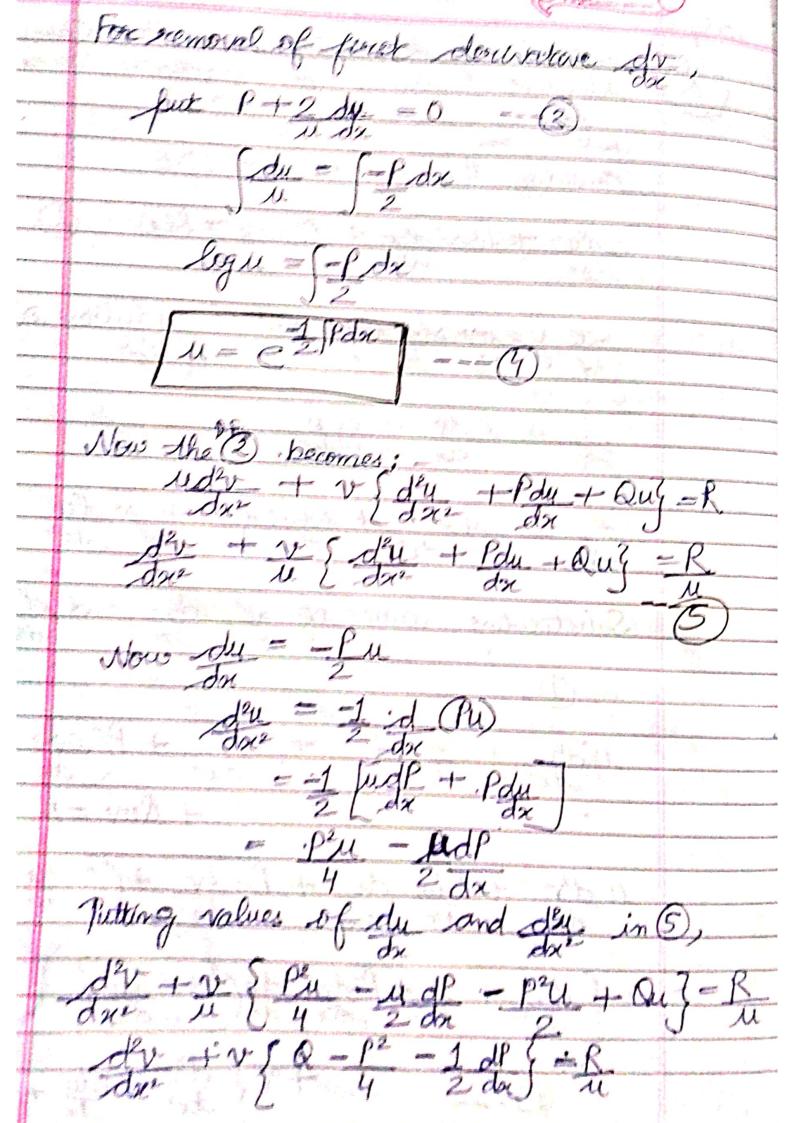


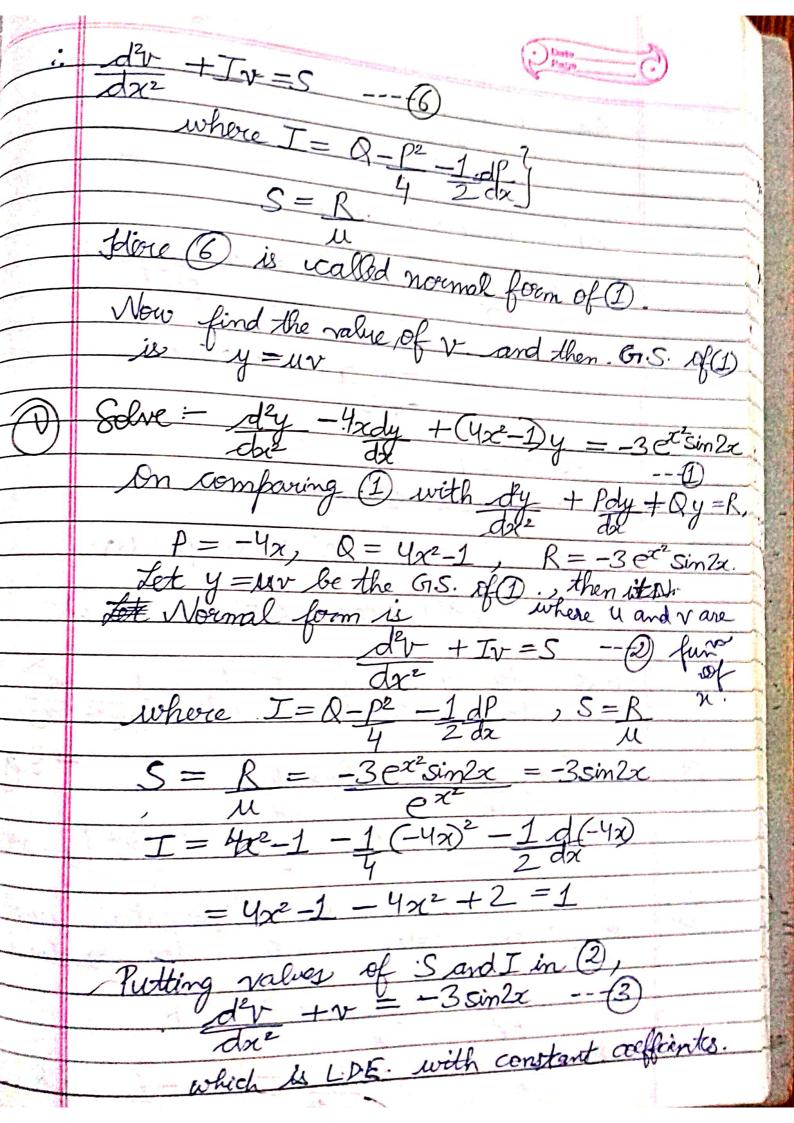


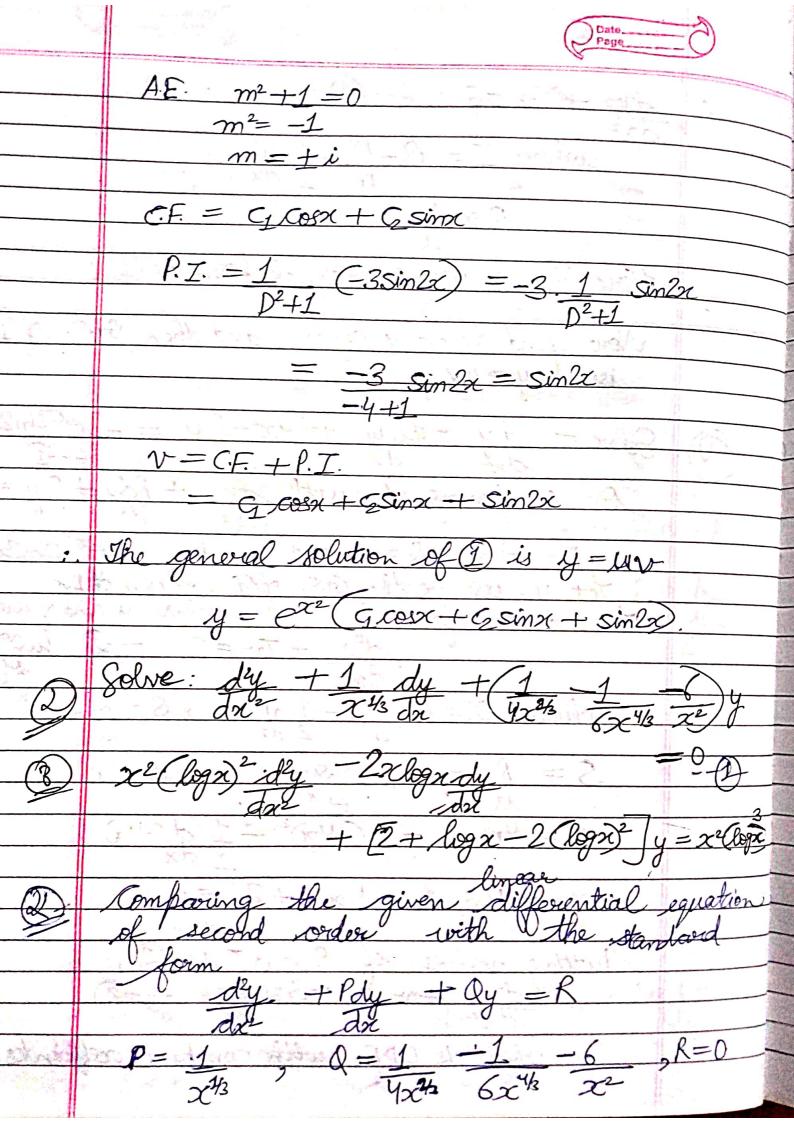


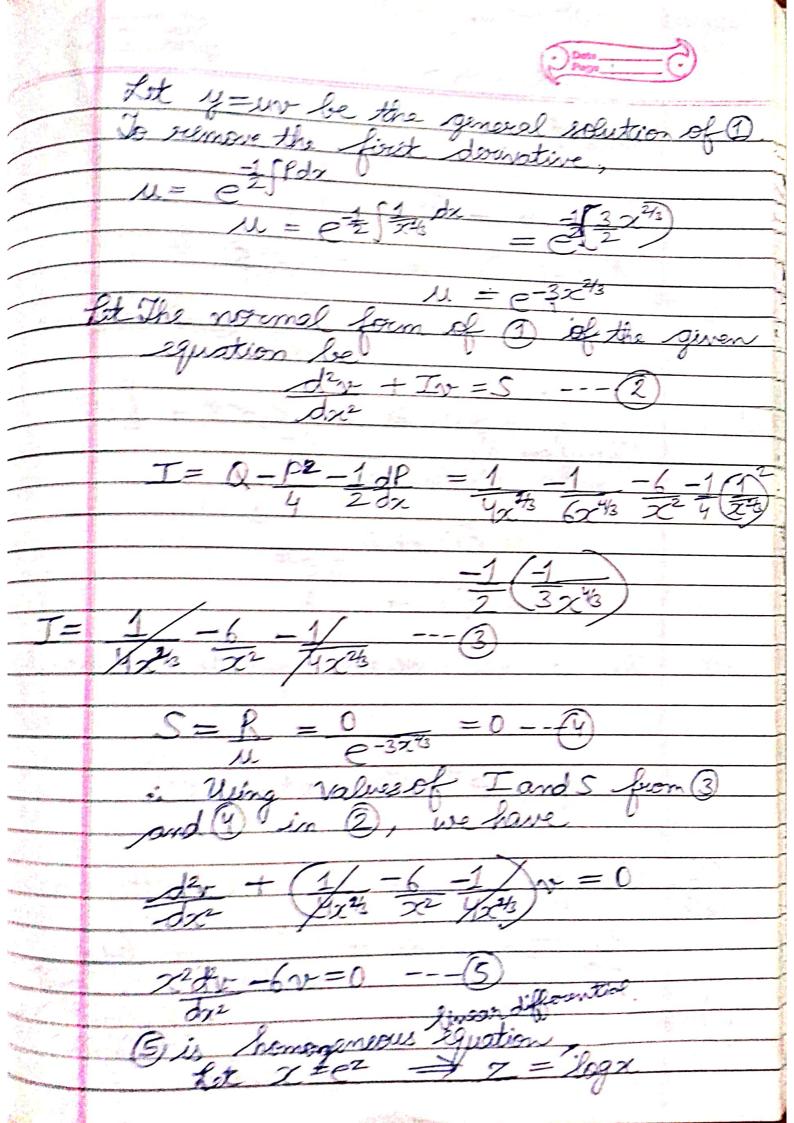
Spirada =  $x^2(-10^{23})$  -  $(3x^2)$  -  $\sin(3x)$ I function is physical and complish fixing V = x en + ex + gex + gex + gexV =  $(x+g)e^{x} - e^{x} + gex$ Testing the value of x + in(2), we have  $y = (x^2+gx)e^{x} - xex + xg$ exhich is the required gameral solution of gamera DE.













$$D \equiv \frac{d}{dz}$$

$$(D^2-D-6) v=0$$
 --- 6

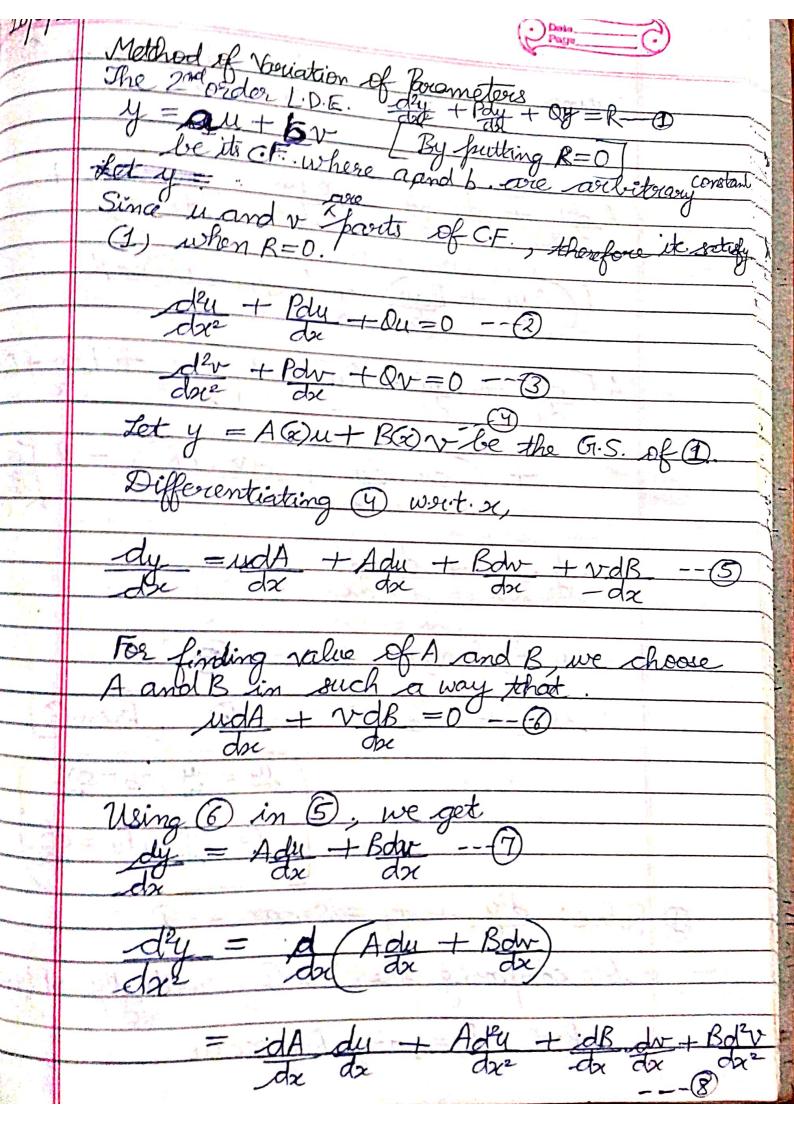
$$m = -2,3$$
  
C.F. its  $V = Ge^{-2z} + Ge^{3z}$   
Therefore the P.I. = 0  
Solution of 5 is

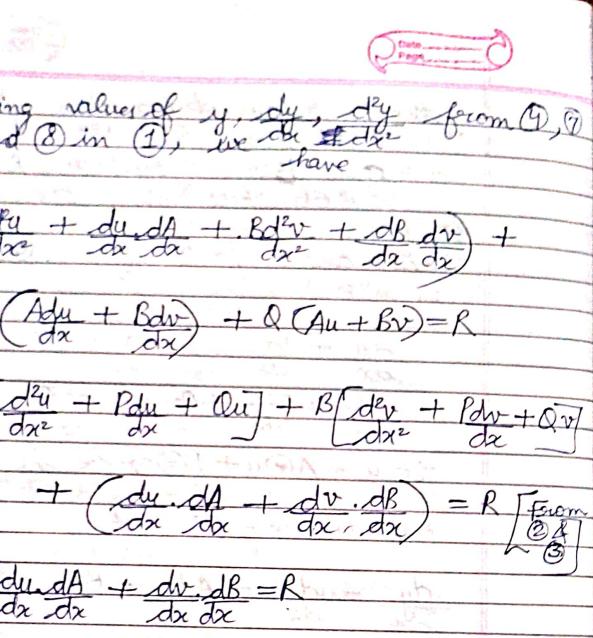
$$V = C.F. + P.I.$$

$$V = Ge^{-2z} + Ge^{3z}$$

$$V = Gx^{-2} + Gx^{-2}$$

$$y = e^{-34}\chi^{2/3}(gx^2 + gx^3)$$





+ dv.dB

 $\frac{dv.dB}{dx} = R$ 

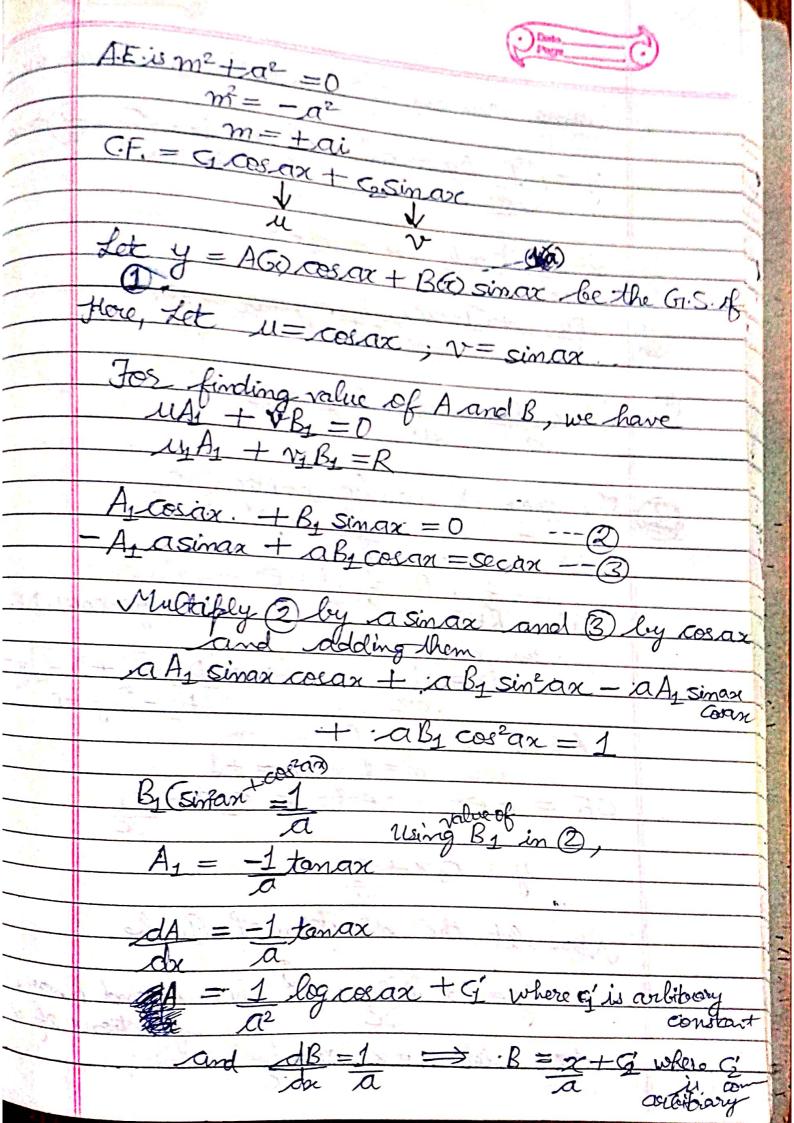
du + Bdv

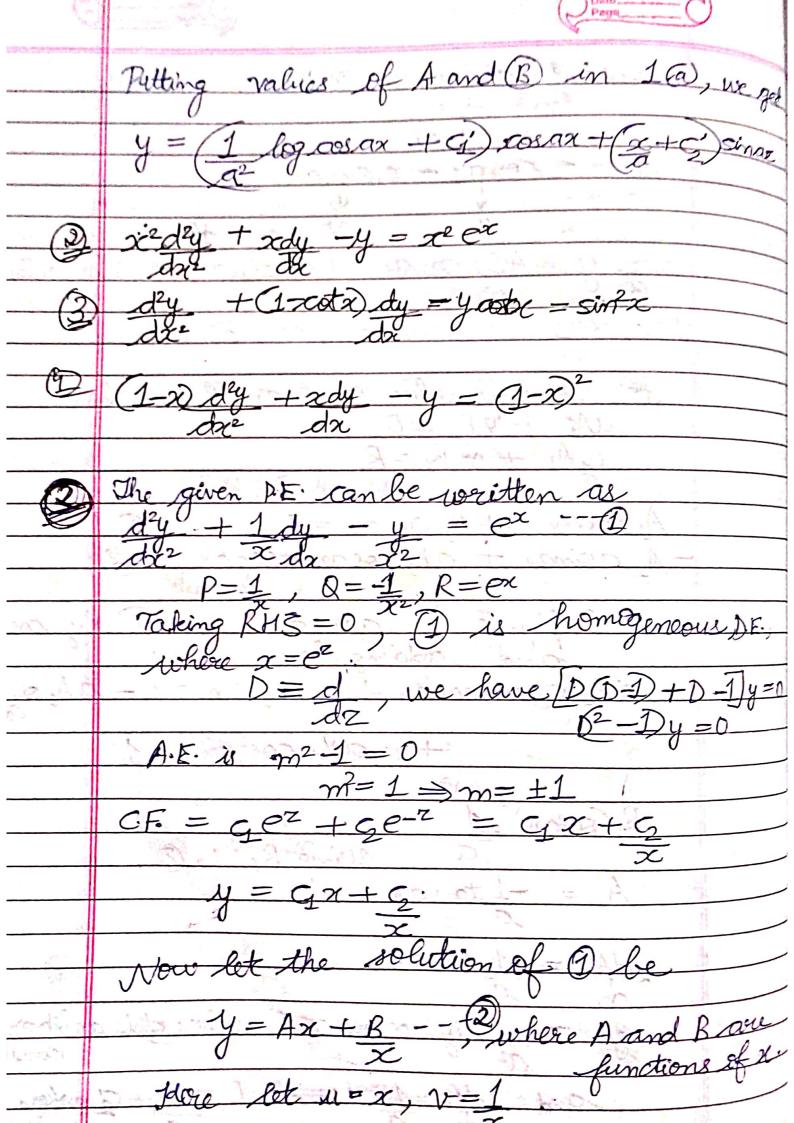
1 + VBy = 0

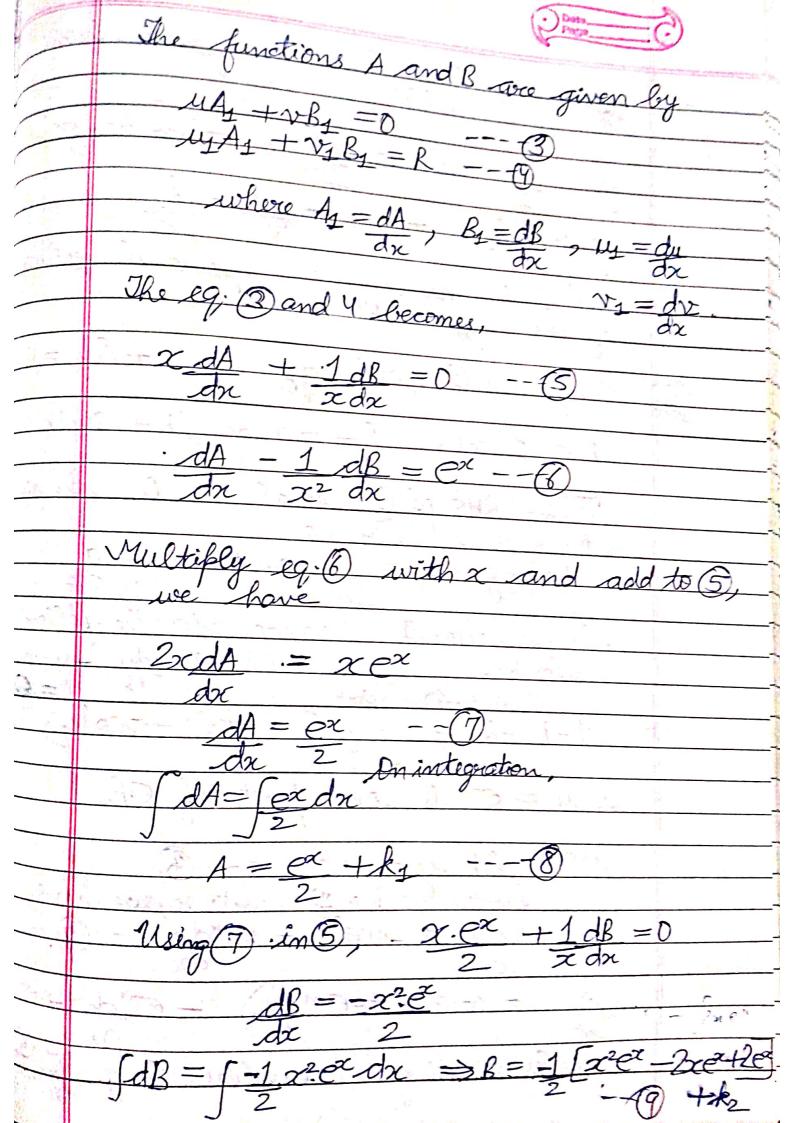
 $M_1 = \frac{du}{dx}$ ,  $v_1 = \frac{dv}{dx}$  $A_1 = dA$ ,  $B_1 = dB$ 

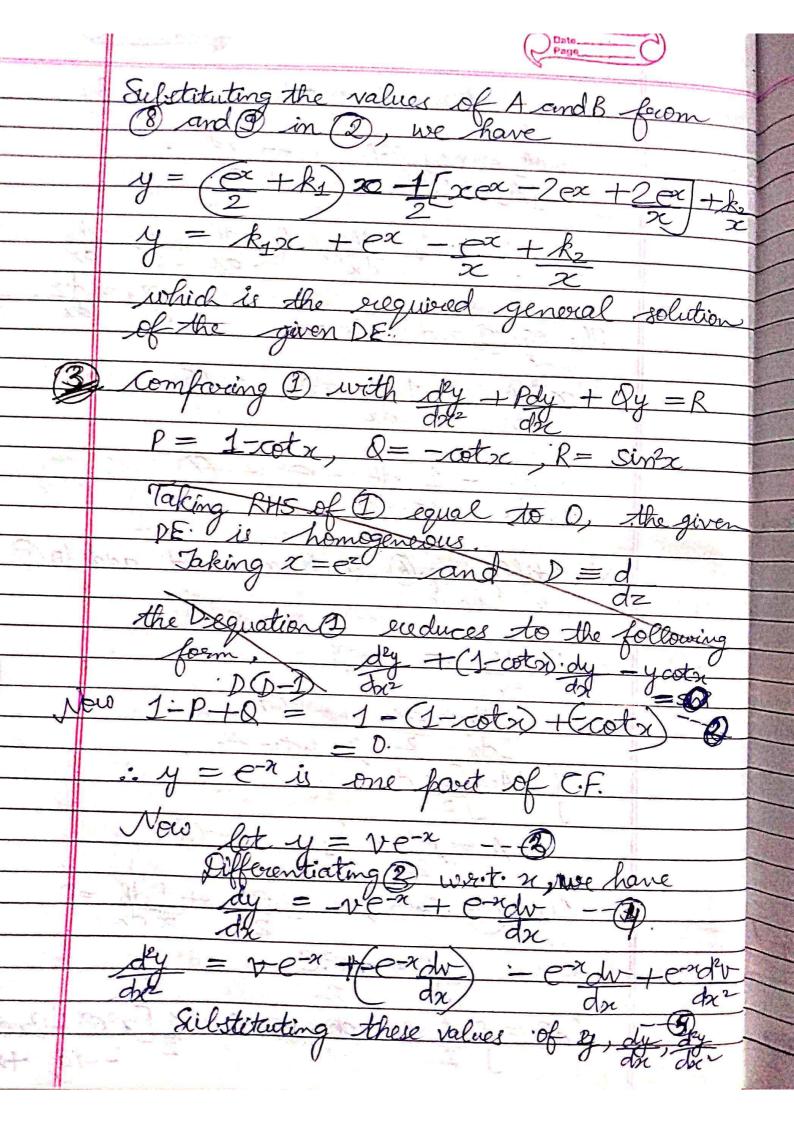
comparing the D.E. D with dry + Pdy + Ry

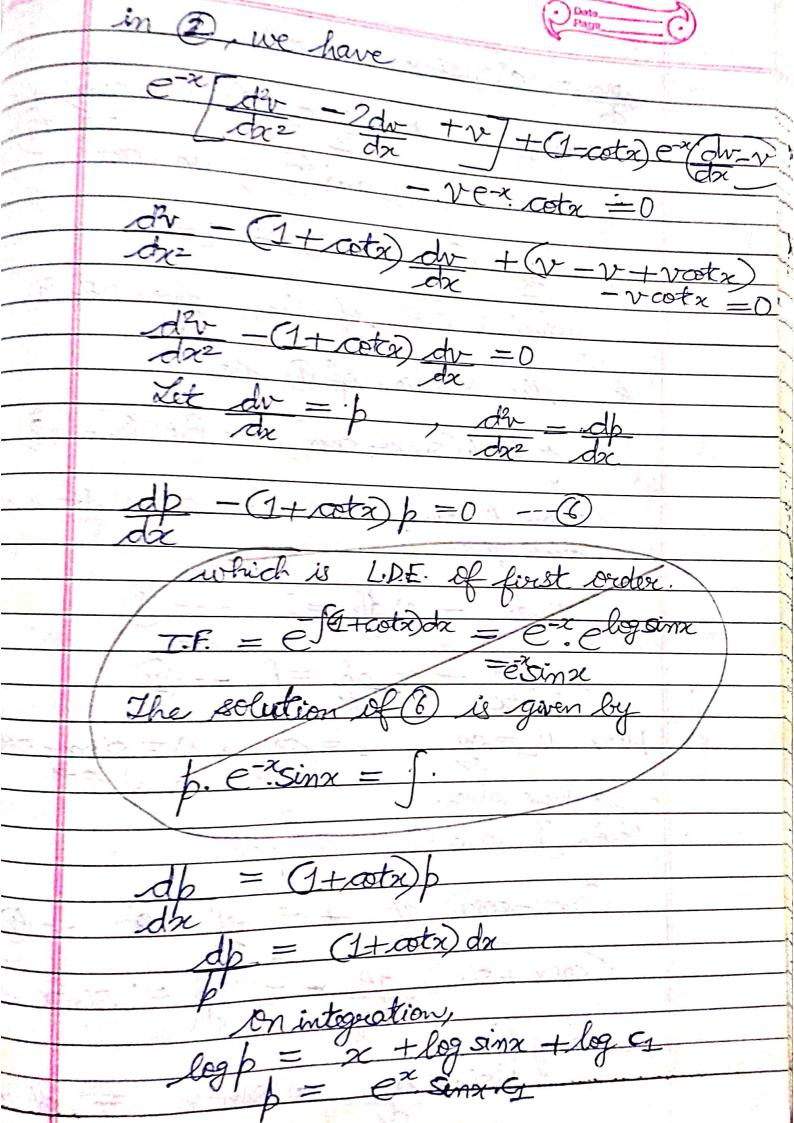
= a2, R = secax

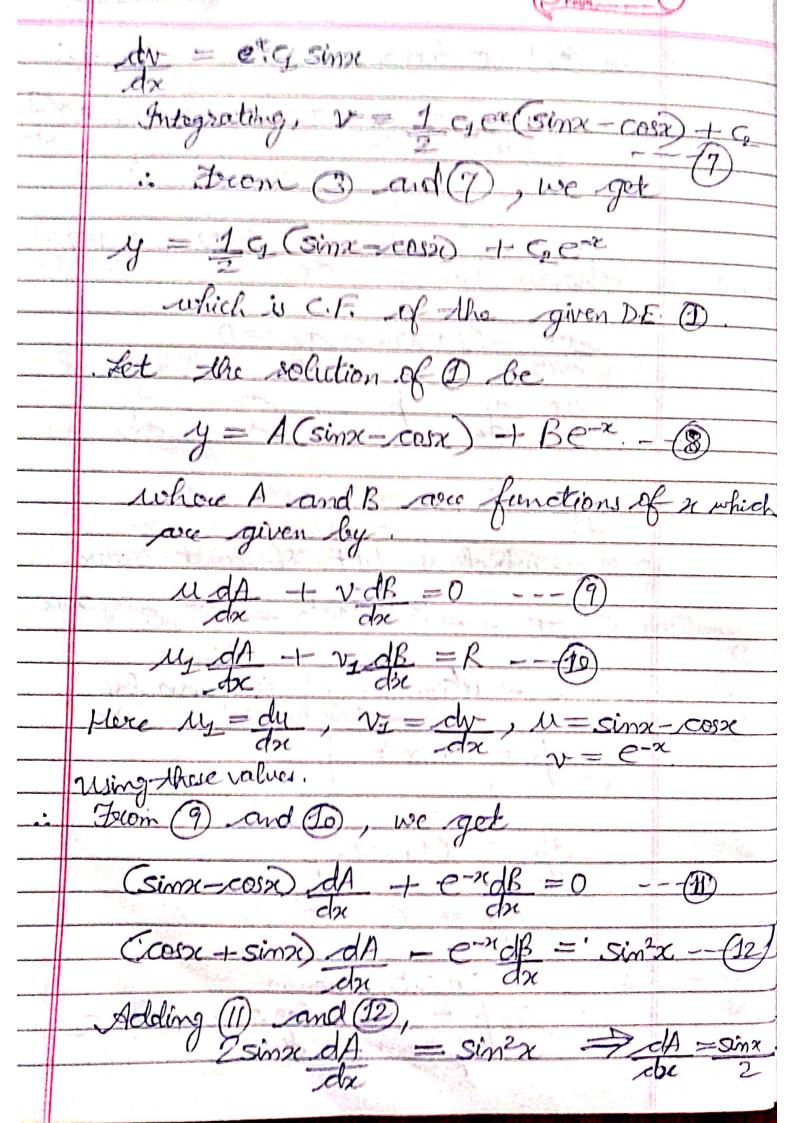


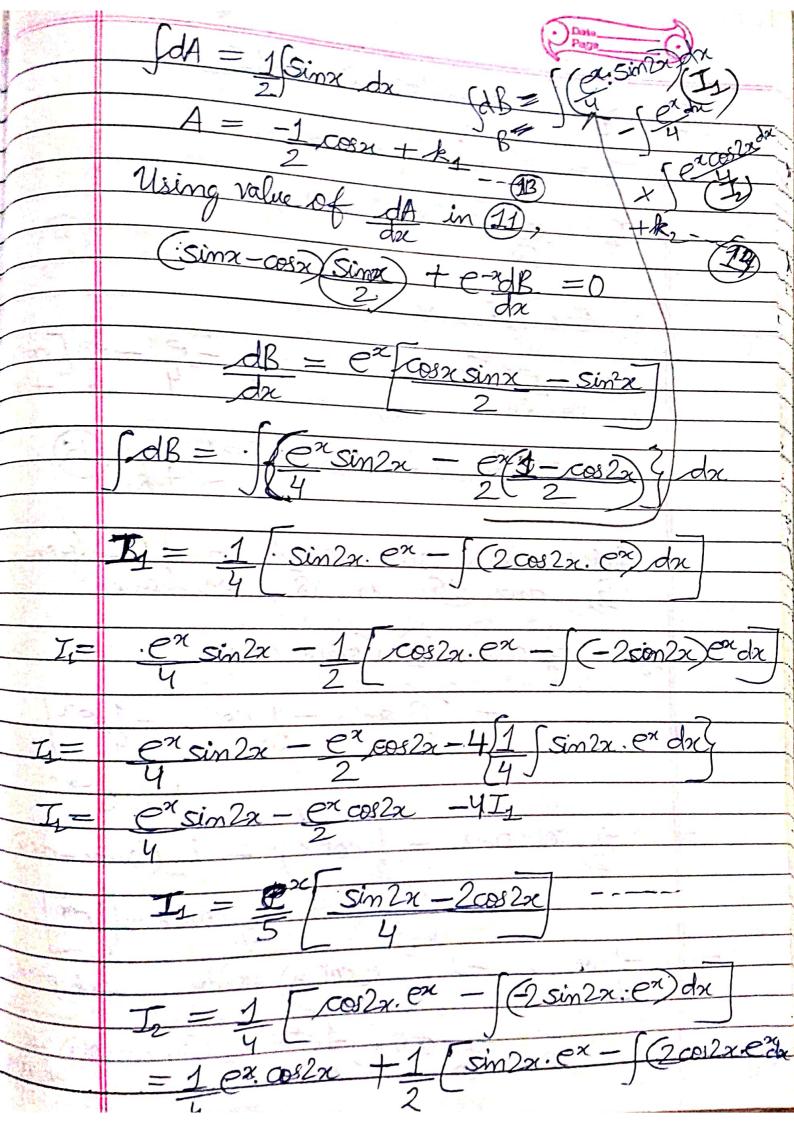


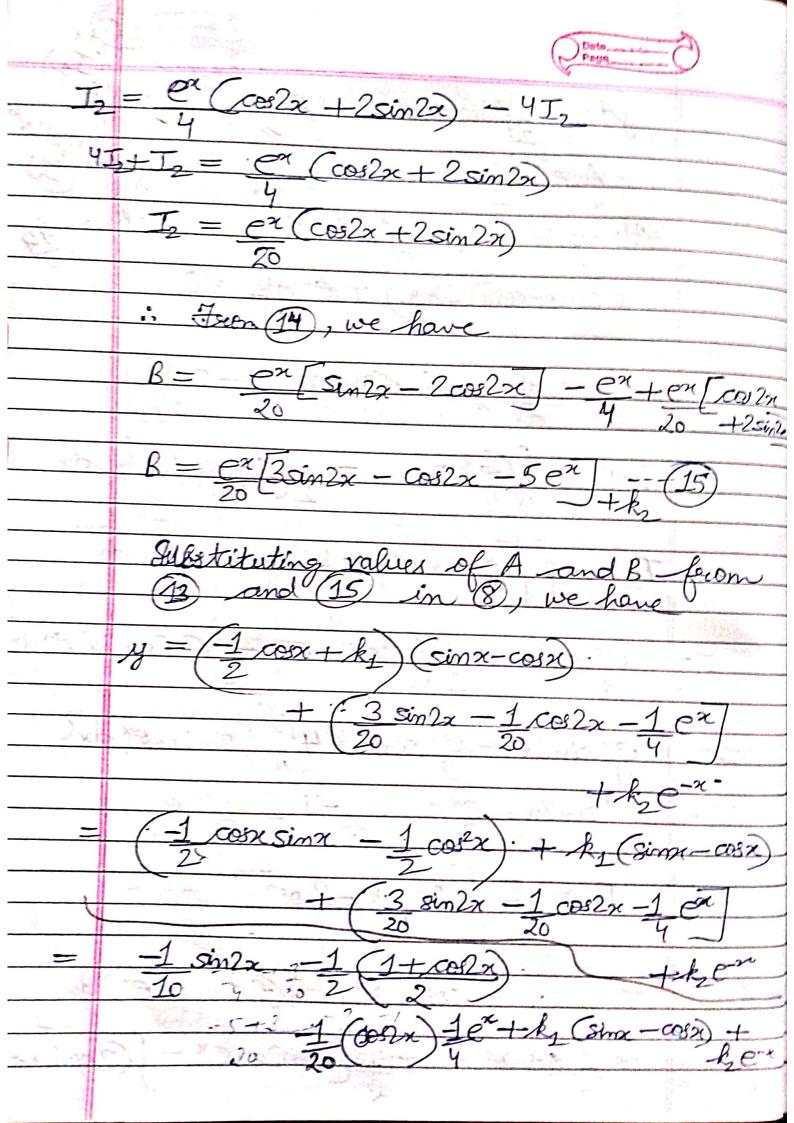


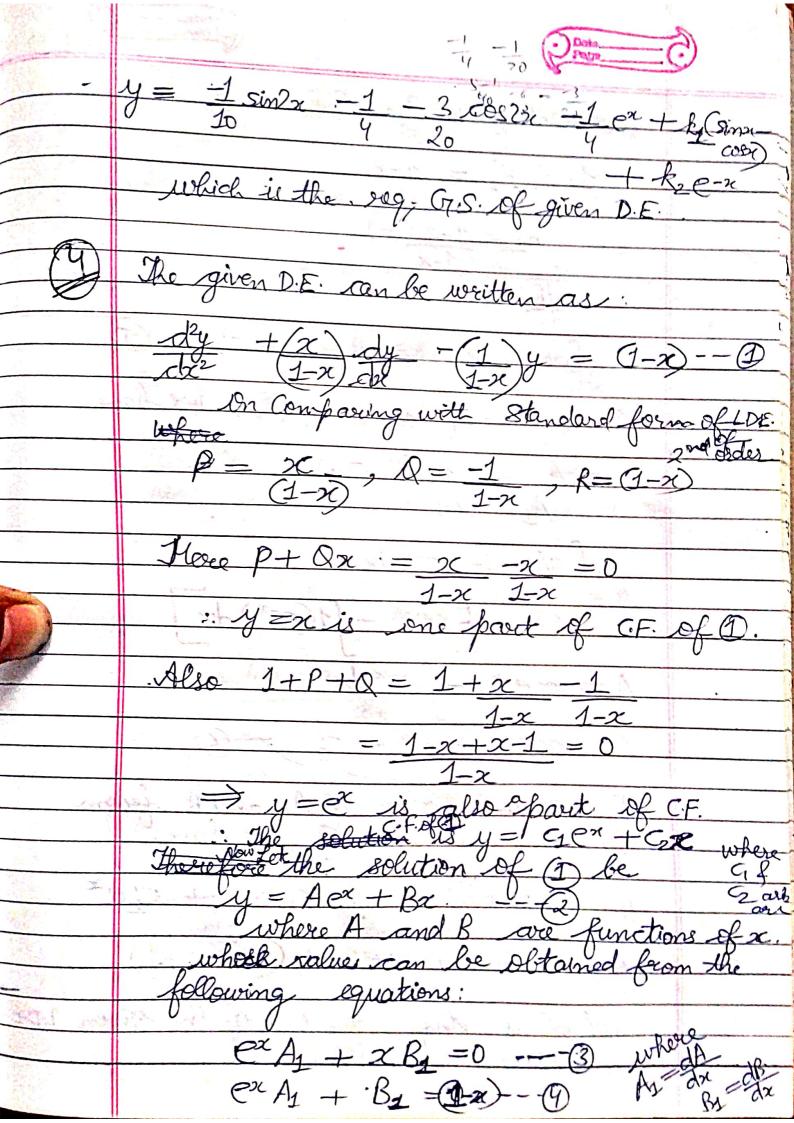


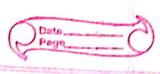












Subtracting 3 fewn 9, we get

 $(1-x)B_1 = 1-x$ 

 $\beta_1 = 1 - x = 1$  1 - x

dB =1

 $\int dB = \int dx \implies B = x + C_1' - - G$ 

Using value of By in eq 3), we have

 $e^{x}A_{1} + x(1) = 0$ 

 $A_1 = -xe^{-x}$ 

 $\int dA = \int -xe^{-x} dx$ 

 $A = -\int xe^{-x} - \int 1(e^{-x}) dx$ 

 $A = \chi e^{-\chi} \cdot - \left[ e^{-\chi} d\chi \right]$ 

 $A = xe^{-x} + e^{-x} + c_2' - -6$ 

Substituting values of A and B ferom (5) and (6) in (2), we have

 $y = (x+1) + (x+g)x + c_2e^x$ 

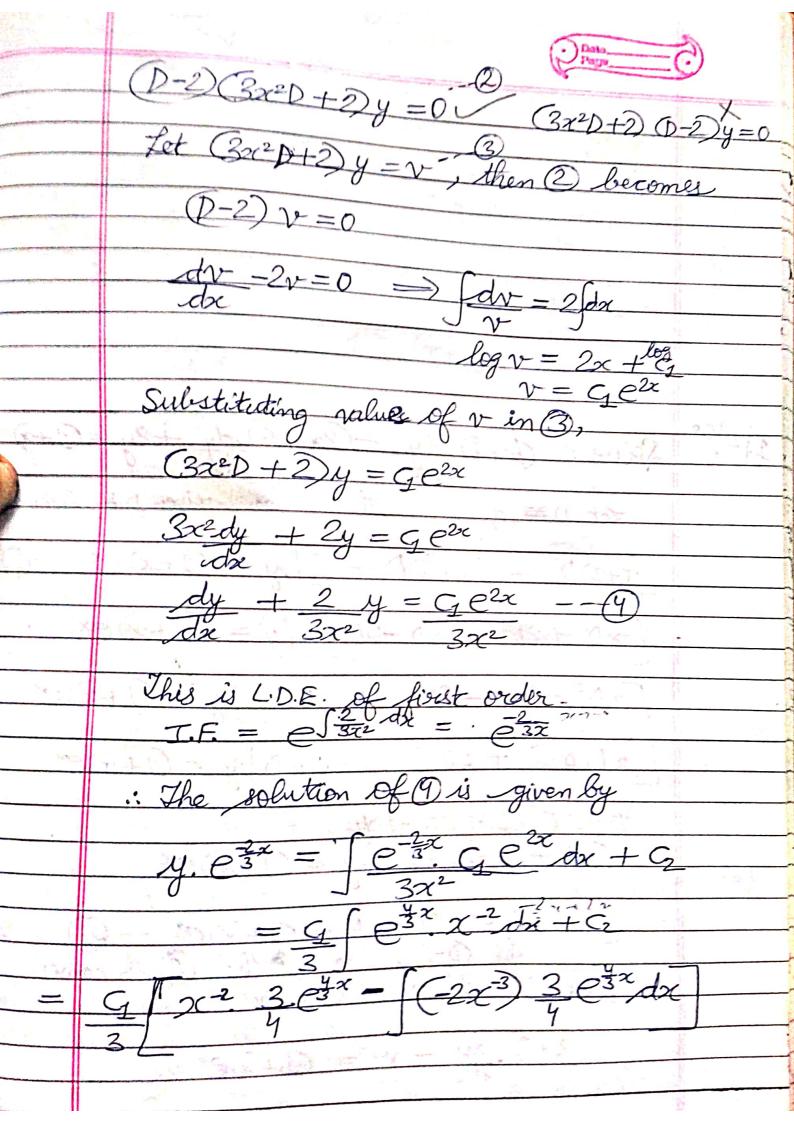
 $y = Gx + Ge^{x} + (x + x + 1)$ 

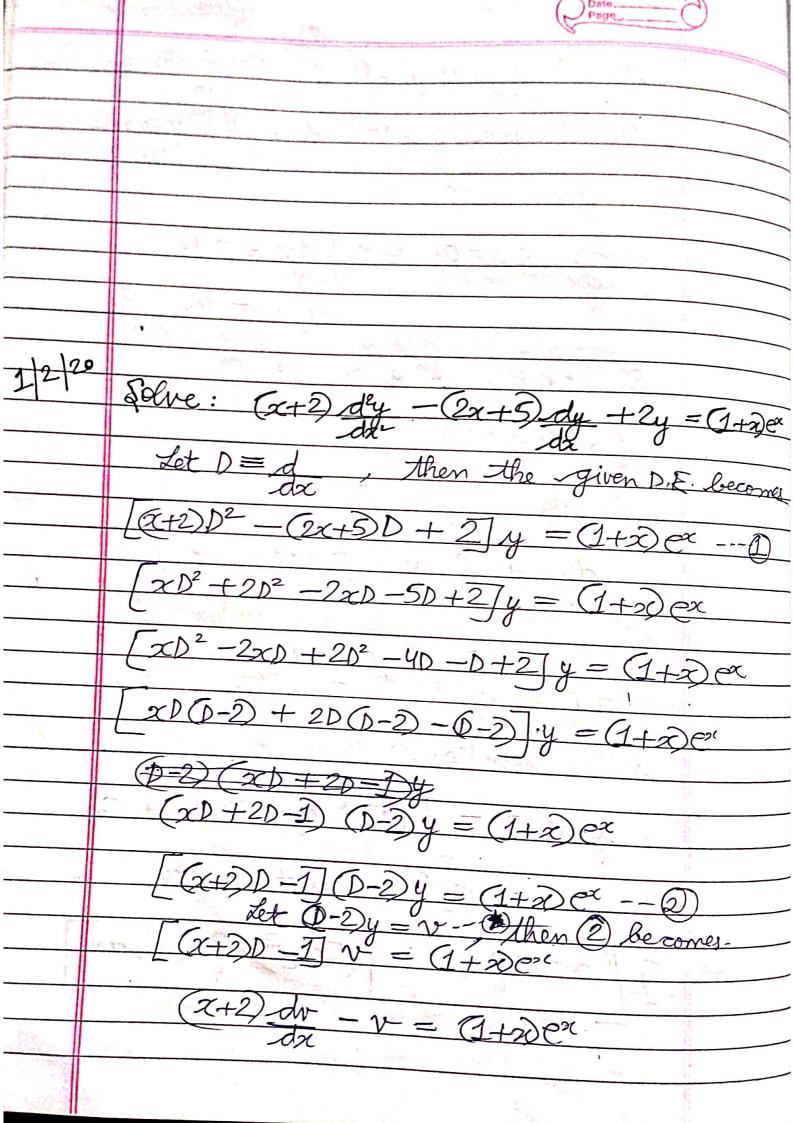
which is the seg. Gr.S. of the given LD.E.

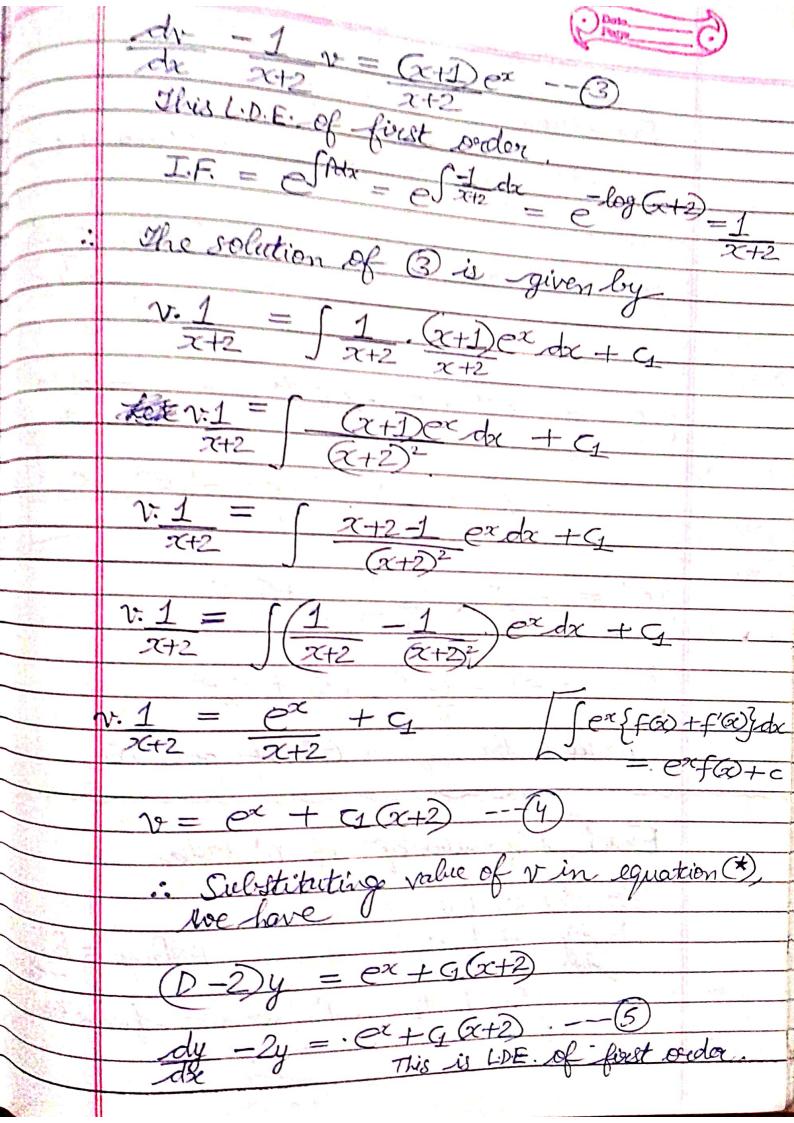
a solve: - 35thy + (2+6x-6x2) dy -4y=0 " street of sprinteral factors of w Then the given DE. can be written as 3x20° + (2x6x-6x2)0-4)4=0 D(3xb+2)-2(3x20+2) 3x20° + (2+6000 - 2(3x20+2) | y = 0 14=0

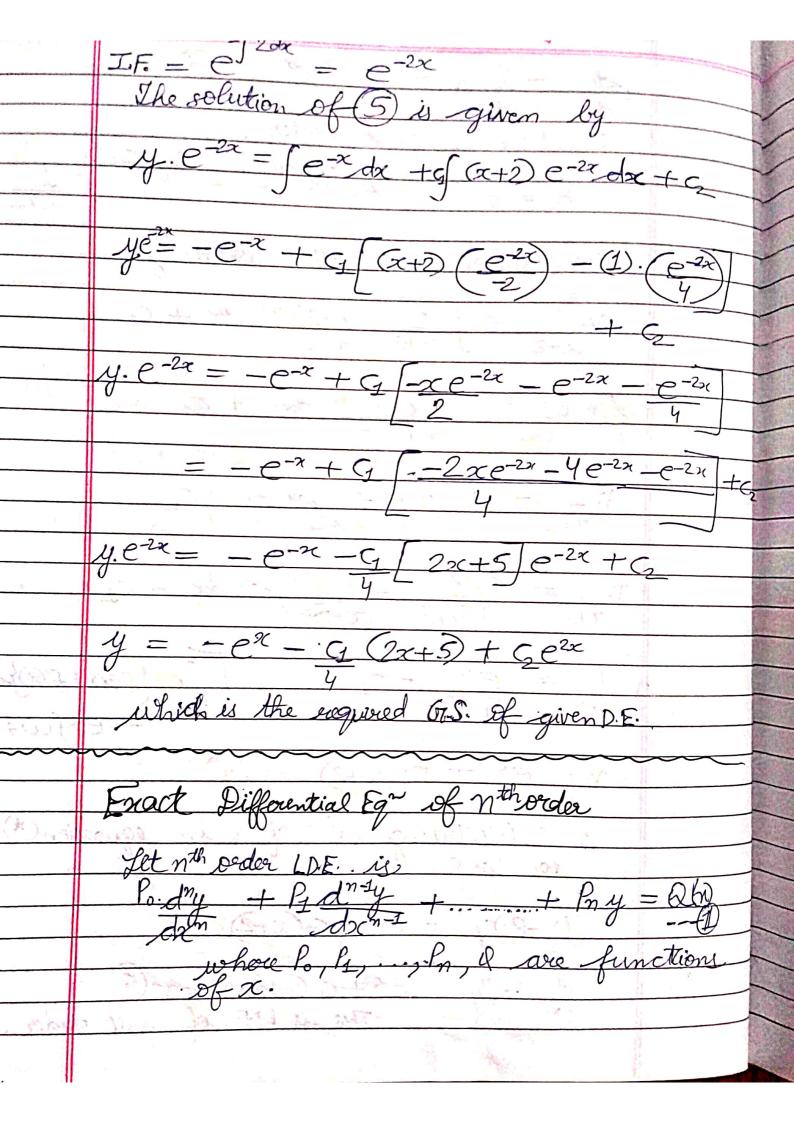
Mothod of Operational factors 7 (Undetermined Coefficients)

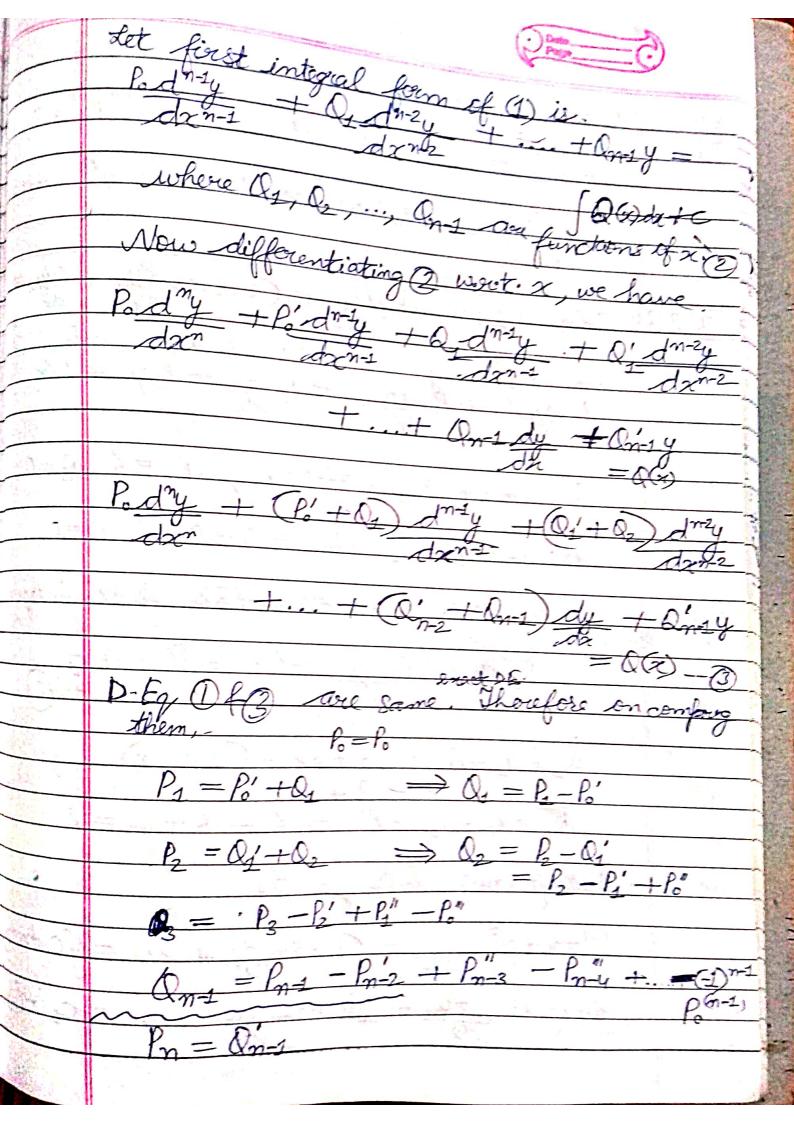
Solve:  $3x^2d^2y + (2+6x-6x^2)dy - 4y = 0$   $dx^2 - dx$ Let D = d dxThen the given Determined coefficients  $[3x^2D^2 + (2+6x-6x^2)D - 4]y = 0$   $[3x^2D^2 + (2+6x)D - 2(3x^2D+2)]y = 0$   $[D(3x^2D+2) - 2(3x^2D+2)]y = 0$ 

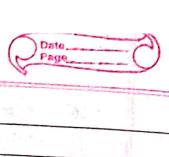












 $\frac{p_n-q_{m-1}}{p_m-q_{m-1}}=0$ 

 $P_n - P_{n-1} + P_{n-2} - P_{n-3} + \dots + (-1)^n P_n^m = 0$ 

. which is exactness condition for nth order D.E. (1).

3 2 20 Solve:  $(1+x+x^2)\frac{d^3y}{dx^2} + (3+6x)\frac{d^2y}{dx^2} + 6dy = 0$ On Comparing given D.E. equation with -- (1)  $P_0 \frac{d^3y}{dx^3} + P_1 \frac{d^2y}{dx} + P_2 \frac{dy}{dx} + P_3 y = Q(x)$   $\frac{dx^3}{dx^3} = \frac{dx^2}{dx}$ 

 $P_6 = 1 + x + x^2$ ;  $P_1 = 3 + 6x$ ,  $P_2 = 6$ ,  $P_3 = 0$ 

Now  $P_3 - P_2' + P_1'' - P_0''' = 0 - 0 + 0 - 0 = 0$ 

: Given DF. is exact whose first integral som is given by

lody + Ordy + Ordy = I Odx + 9

dx2 dx -- 2

 $Q_1 = f_1 - f_0' = 3 + 6x - (1 + 2x)$ = 2 + 4x

 $A_2 = P_2 - P_1' + P_0'' = 6 - 6 + 2 = 2.$ 

Putting values in 2), we get

(1+x+x2) dy + (2+4x) dy + 2y = fodx+g

(1+x+x2) dy + (2+4x) dy +2y = Cy-63)

