Differential Equations

UNIT- III and IV

Ву

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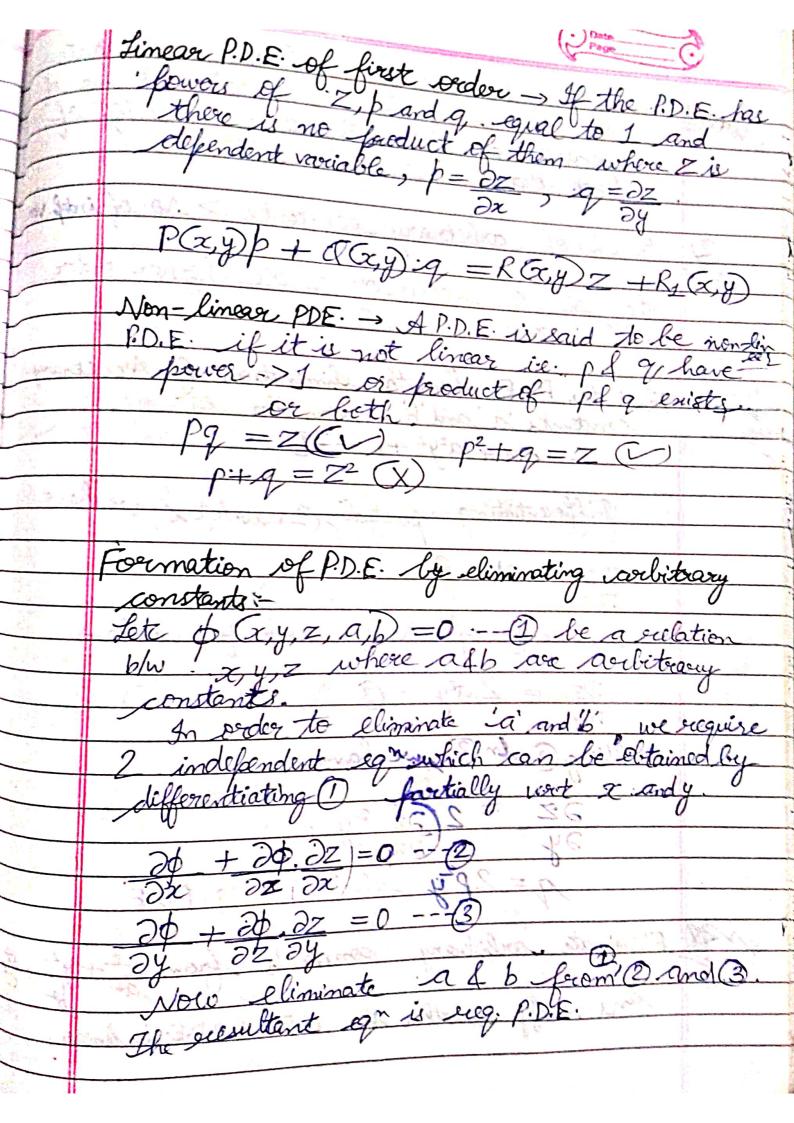
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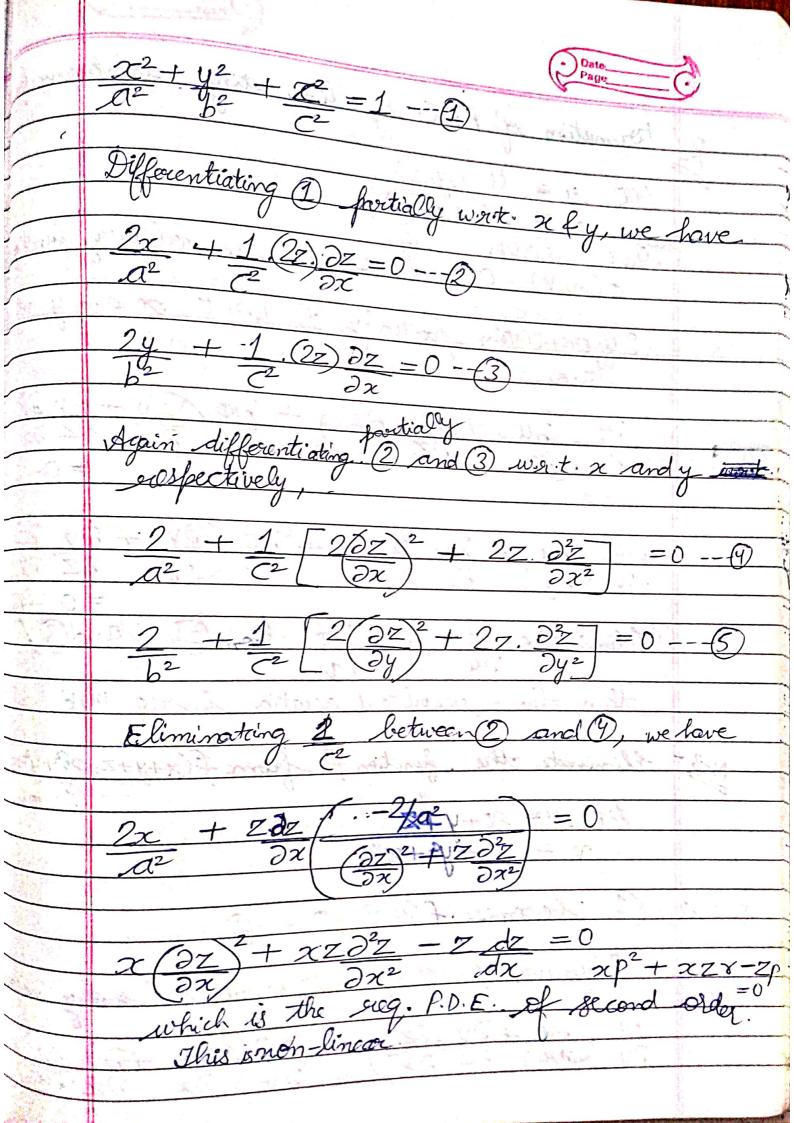


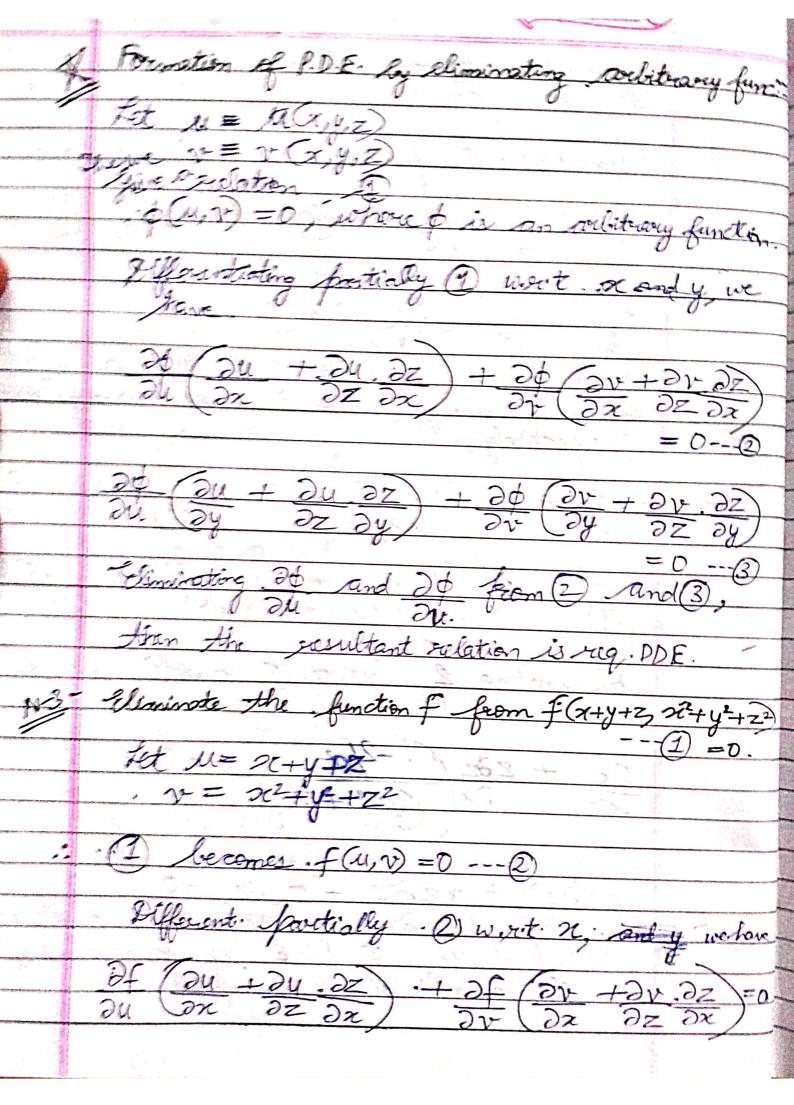
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2/1/0-	Octial D.F.
4/2/20	Partial D.E.
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	in the represents 310 lotter 100
	De dependent raviale. I il
2 - 7	10. ret. 1 della de la la descritiva
	D.E An egr jobich reforesents relationship W. defendent raviable & its derintive W. ret. Independent variable
	A.D.E
1	O.D.E. P.D.E.
	the state of the s
	Arasi Limona P.D.E al a 1 -1 66
	first order Jagranery with
	Duasi-Linear P.D.E. of first order (Logranges 14)
	$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, $r = \frac{\partial z}{\partial x^2}$
	$\frac{1}{2}$
	∂x^2

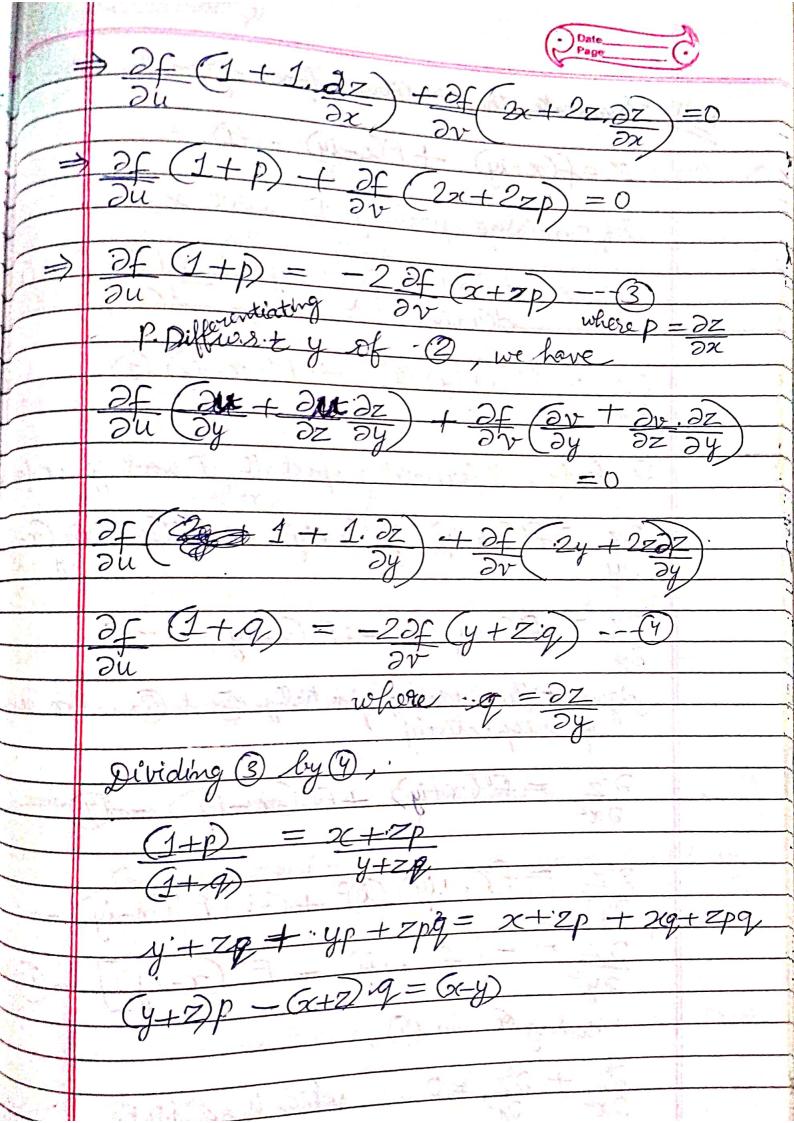
	$S = \frac{\partial z}{\partial x \partial y} \qquad t = \frac{\partial^2 z}{\partial y^2} \qquad old less the plane of $
10 110 -	2224
1	John John State Of
-3.2	A. P.C.
	A P.D.E. is soid to be grasi-linear P.D.E. of it is Clinear in p and 9 and 2 may be non-linear.
	it is linear in and a
	be mon-linear
	o made.
	F(x,y,z) = Z + QG(x,y,z) = R(x,y,z) = R(x,y,z)
	$P(x,y,z) \frac{\partial z}{\partial x} + Q(x,y,z) \frac{\partial z}{\partial y} = R(x,y,z)$
4	
1	
-)00	$xyzy + xyzyq = x^2y^2$
- 1	b-1
Sinon	
2 0	Service Con
	pq + q = xyz (x)
+ 1/2	sins of the same
	0 . 0
	Semi-linear or almost linear P.D.E.
	A P.D.F. of the form P(x,y) b +Q(x,y) a
	p to the
1	= P
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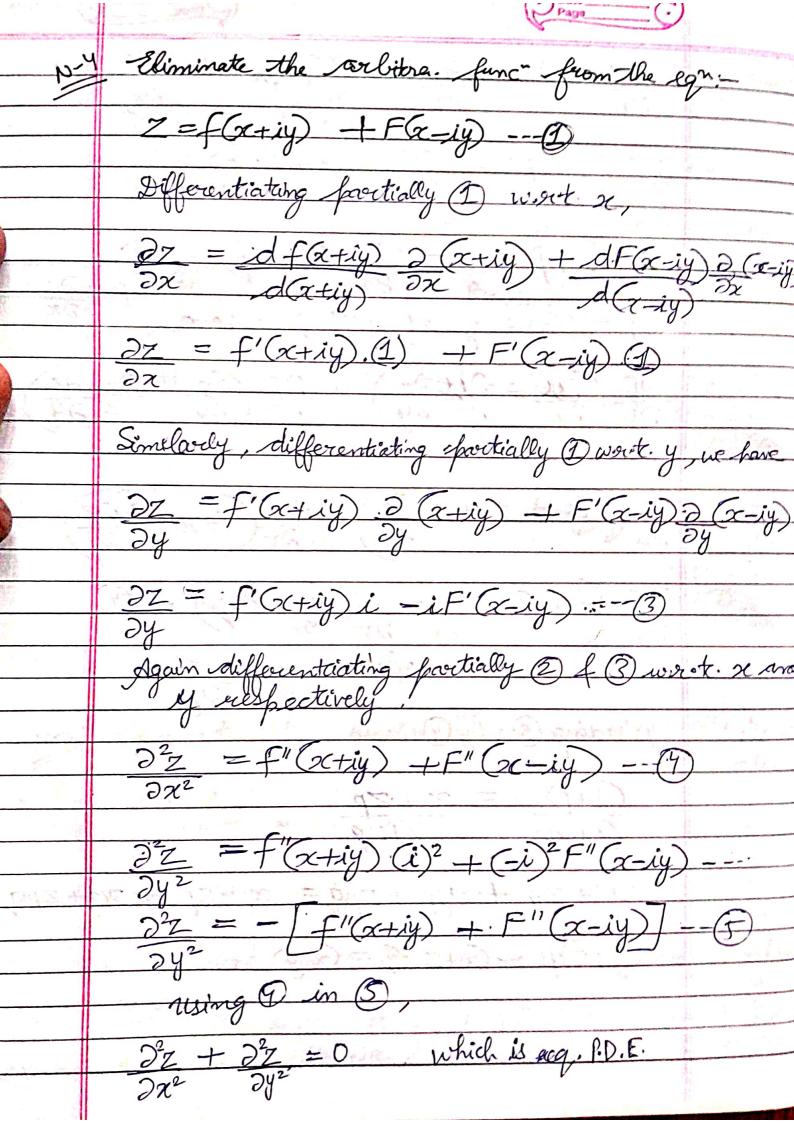


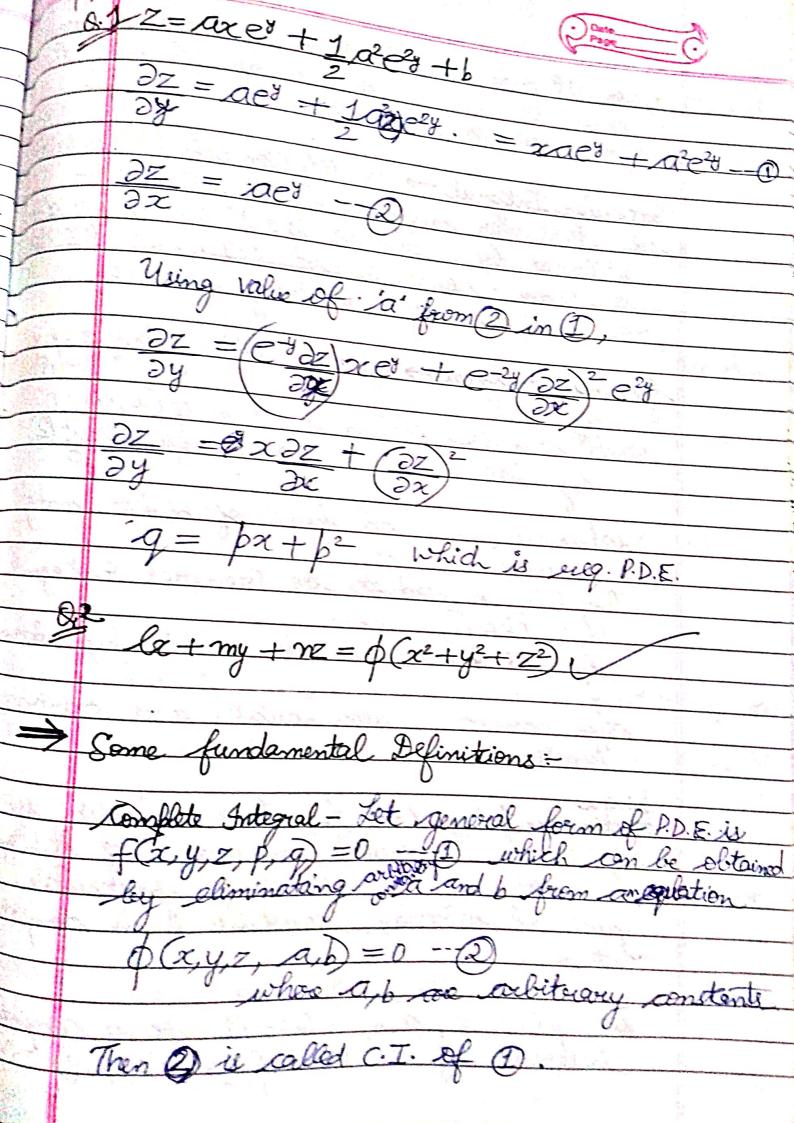
Note In eq. (1), the no. of substrany constants to be eliminated is just equal to no. of independent variables, and thus an eq. of first order. B If No. of arbitrary constants > No. of inthe variable the eq" of second order or higher order will asise Find the P.D.E. by the elimination of Arbitrary
constants a and b from the eg. $Z = ax + a^2y^2 + b - D$ Differentiating factially (1) wrotz, we have 37 = 10 - 10 m Again diff Ofwritially write y, we have $\partial z = 2a^2y - 3$ Ming 2 dos 3) que have $\partial Z = 2(\partial z)^{2}y$. $\partial y = 2\rho^{2}y$ ohich is reg. P.D.E. Eliminate voliterry constants from x2+42-ti= and voily whether the obtained P.D.E. is linear.









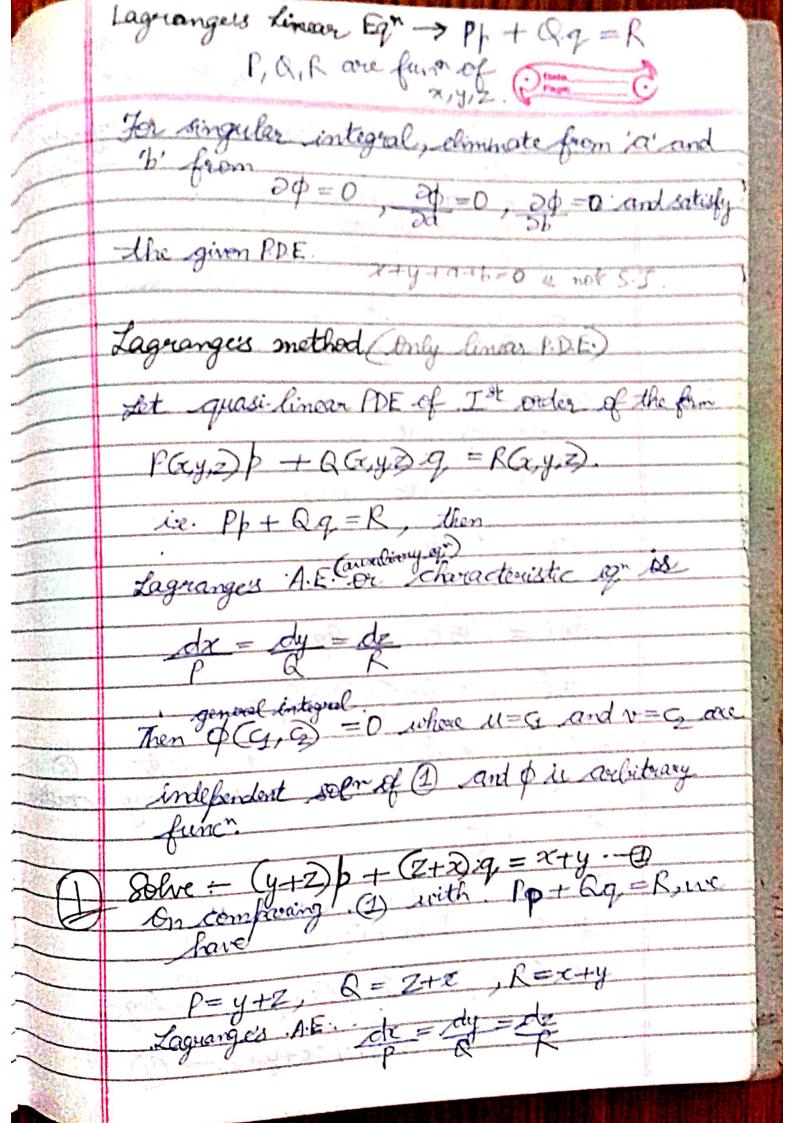


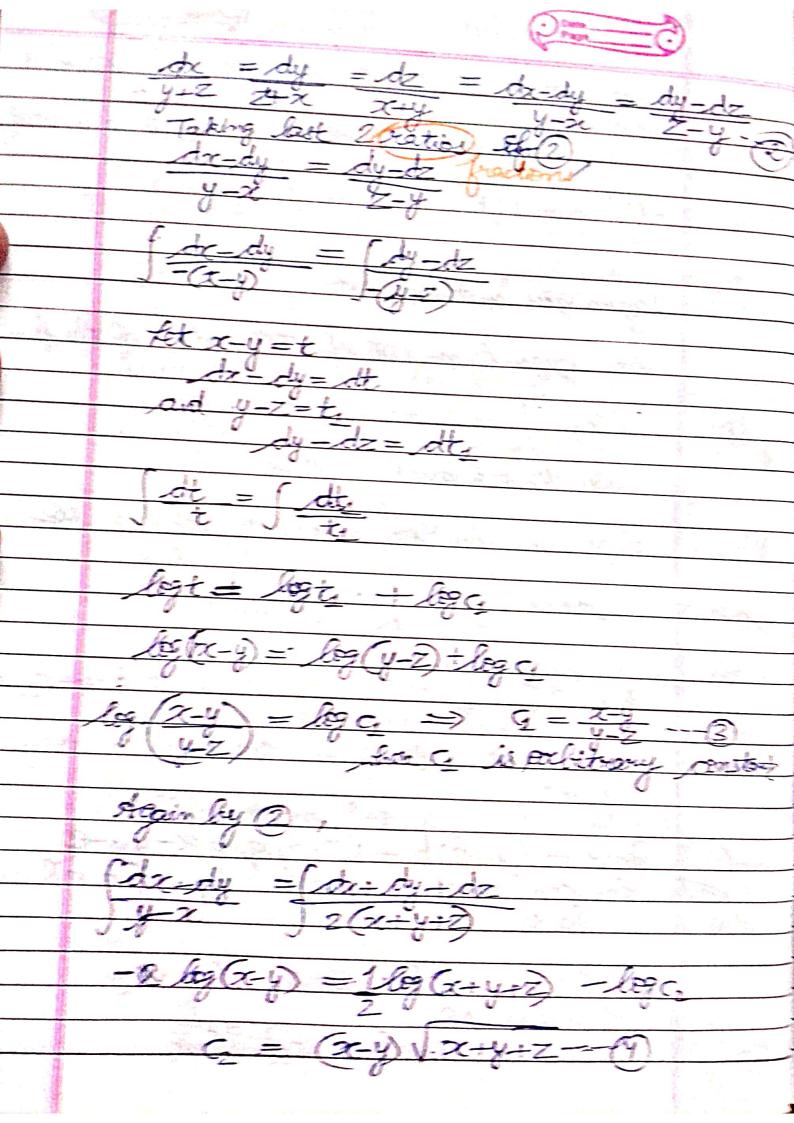


The solution is said to be CI Particular Integral -> A solution of PDE is said to be forticular integral if it can be obtained by particular values of particular values of antituory constants in C.T. ic. P.I. clocs not constants any auditory constants. General Integral - Let the PDE (Goy, Z, P, 9)=0 which can be obtained from \$(u,v)=0.
Loy eliminating eviloitowy function (2) where it and v are funct of x, y, z and of is arbitrary funct.

Then (2) is said to be Greneral Integral of P.D.E. (1) it.

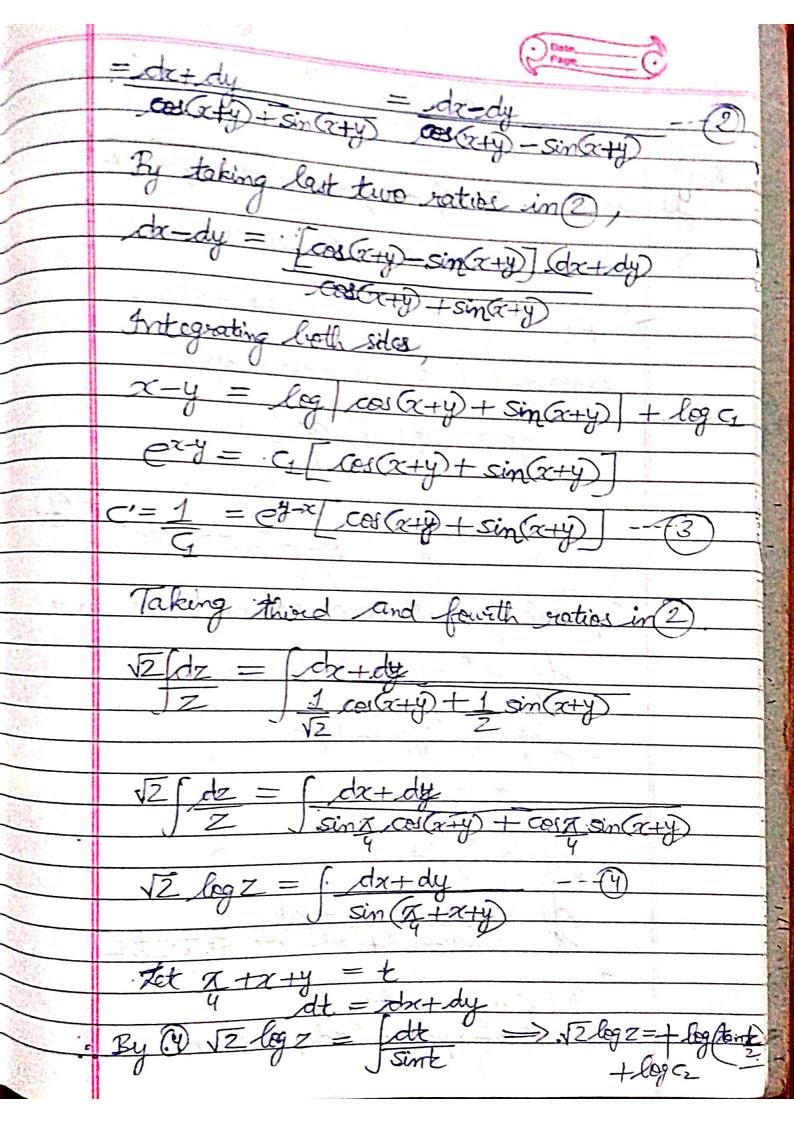
Lyeneral Integral is the relationship Retween defen soviable, indefer reviable and corbitorary Forticular Antegral -> A solution of P.D.E. is said to be singular integral if it cannot be obtained by futting facticular plues of axbitrary constants in its compute integral, for $\phi(x,y,z,a,b) = 0$ -- 1) be a complete integral of the PD.E. f(x,y,z,p,q) = 0-0

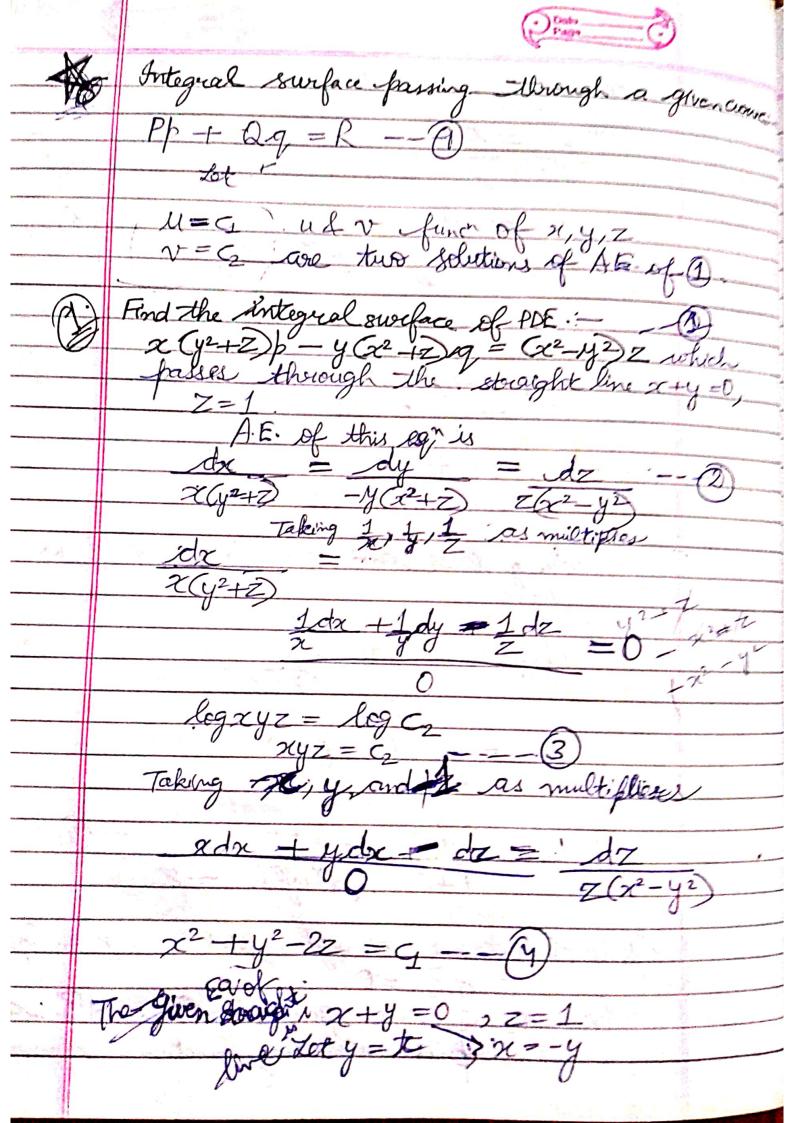




* General Integral of givenPDE is given by $\phi \left(\frac{x-y}{y-z}, (x-y)\sqrt{x+y+z}\right) = 0$ is the rug G.S. Solve: (y2+22-22)p-2xyq-+2xz=0-1) The given P.D.E. con be written as $(y^2+7^2-x^2)^{\frac{1}{p}}-2xyq=-2xz=0$ On compaing (1) with Pp + Qq = R, $P = y^2 + z^2 - x^2$, Q = -2xy; R = -2xzNow, Lagrangers A.E. is given by dx = dy = de $\frac{dx}{y^{2}+z^{2}-x^{2}} = \frac{dy}{-2xy} = \frac{dz}{-2xz} - \frac{2}{2}$ Taking last two ratios in @, we have $\log y = \log 7 + \log c_1$ y = 9

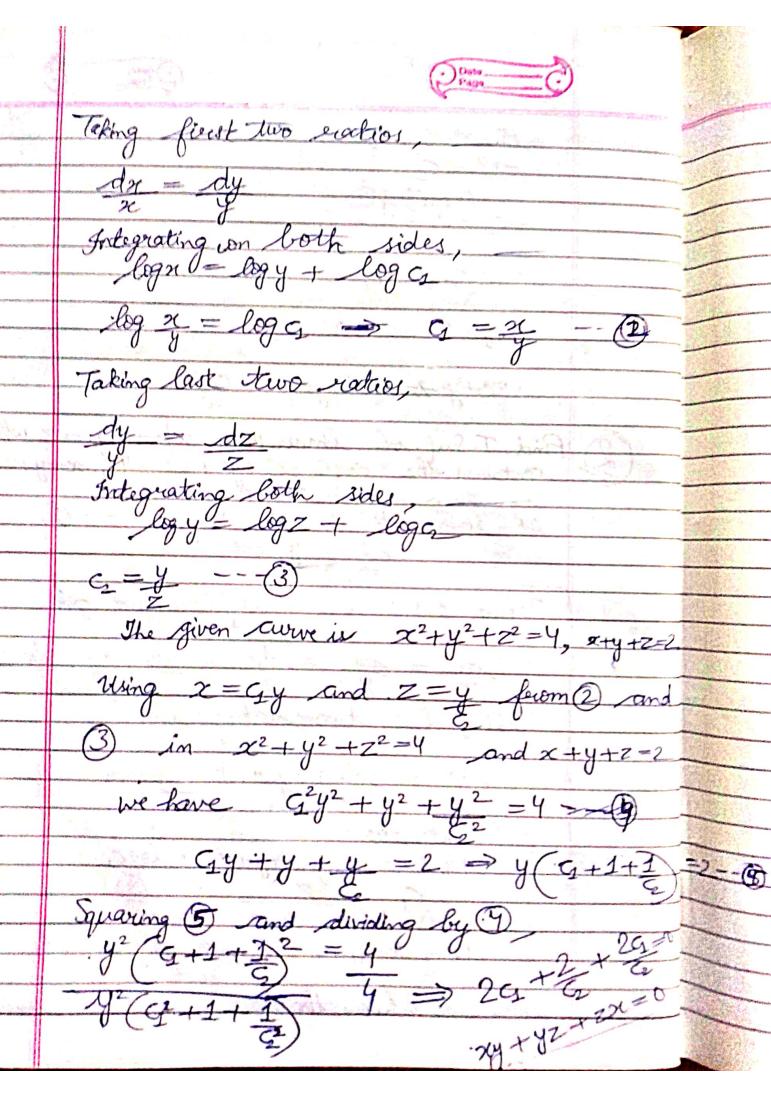
Again by (2), $\frac{dx}{dx} = \frac{dy}{dx} = \frac{dz}{dx} = xdx + ydy + zdz$ $\frac{dy}{dx} = \frac{dy}{dx} = \frac{dz}{dx} = xdx + ydy + zdz$ $\frac{dy}{dx} = \frac{dy}{dx} = \frac{dz}{dx} = xdx + ydy + zdz$ Thing second and fruth ratios in (9) $\frac{dy}{-2xy} = \frac{xdx + ydy + 7dz}{-x(x^2 + y^2 + 7^2)}$ $\int \frac{dy}{y^2} = \frac{2(xdx + ydy + zdz)}{(x^2 + y^2 + z^2)}$ legy = go (2-+y2+22) + log cz $\frac{\lg(4)}{(2^2+y^2+2^2)} = \log c_2$ $y(x^2+y^2+z^2)=c, ---(5)$: General integral of given P.D.E. is given by $\phi(G_1,G_2)=0$ tohere of and of one given by 3 and 5. Solve: pros (x-ty) + 9,5m(x+y)=z -Lagrange's Figuation $\frac{dx}{ce(G+y)} = \frac{dy}{\sin(G+y)} = \frac{dz}{z}$

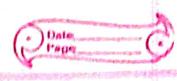




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From G & D C 202-9 G+2C+2=0 22-142-1- 2xyz+2=0 Find I. Surf. of linear P.D.E. xp+yq=z which contains the circle $x^2+y^2+z^2=y$, x+y+z=2. The given eq^x is xp+yq=z. A.E. is dx=dy=dz. Taking 1,1,1 as multipliers, we get $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} = \frac{dx + dy + dz - Q}{x + y + Z}$ Taking last two reation in @ dz = dx + dy + dz Z = x + y + z x + y + z x + y + z x + y + z = 2 in (2), z = 2c x + y + z = 2 in (2), z = 2cNow we know that





Charlitis Method Let non-linear P.D. of Jet gooder be f(ec, y, Z, p, q) = 0 --- D

Then, Charifits A.E. or characteristic eg" is

dx = dy = de = dp Fr F9 Pf, + 9 Fr - (x+pfe)

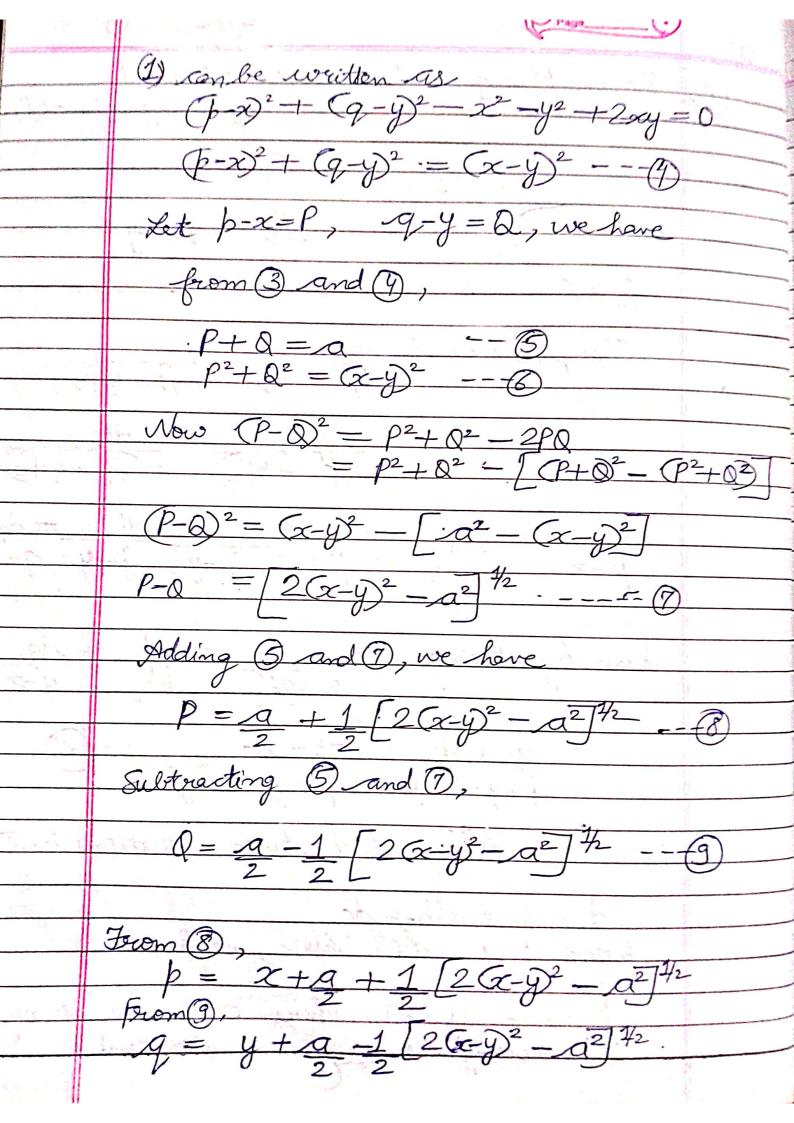
 $\begin{array}{cccc}
&=&dg \\
&-&(f_y+g_1f_2)
\end{array}$ $\frac{dx}{dx} = dy = dz = dp = dg \\
&-&f_R - f_q - p_f_p - g_f_q - f_q + p_f_z - f_q + g_f_z
\end{array}$

Where $f_z = \partial f$, $f_p = \partial f$, $f_y = \partial f$

 $f_q = \frac{\partial f}{\partial q}$, $f_z = \frac{\partial f}{\partial z}$

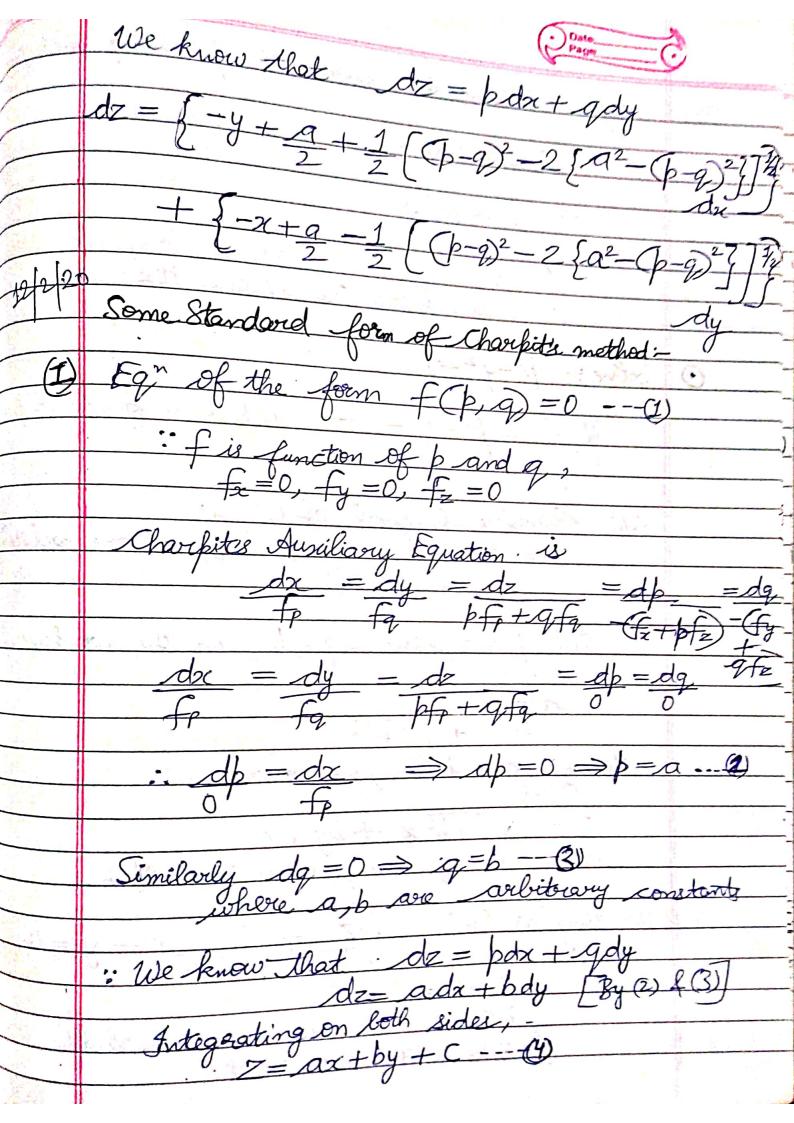
pfq in form of x, y, 2000 C Solve: $b^2 + a^2 - 2bx - 2ay + 2xy = 0$ Let $f = b^2 + a^2 - 2bx - 2ay + 2xy = 0$ $f_{z} = -2p + 2y \qquad f_{p} = 2p - 2x$ $f_y = -2q + 2x$ $f_q = 2q - 2y$ Charpites A.E. is dx = dy = dz $f_p = f_p + qf_q$ $= \underline{db} = \underline{dq}$ $-(f_x + pf_z) - (f_y + qf_z).$ $\frac{dx}{p-x} = \frac{dy}{q^2} = \frac{dz}{p^2+q^2-xp-yq} = \frac{dq}{p^2+q^2-xp-yq} = \frac{dq}{p^2-yq-xq-xq-xq}$ Now by taking $\frac{dx}{p-x} = \frac{dy}{q-x} = \frac{dp}{q-x}$ $= \frac{dx + dy}{p + q - (x + y)} = \frac{dp + dq}{p + q - (x + y)} = \frac{--\sqrt{2}}{p}$ Now, by taking last two relations in @, dx+dy = dp+dq. Integrating on both sides, we have p+q=x+y+a, where a is arbitrary

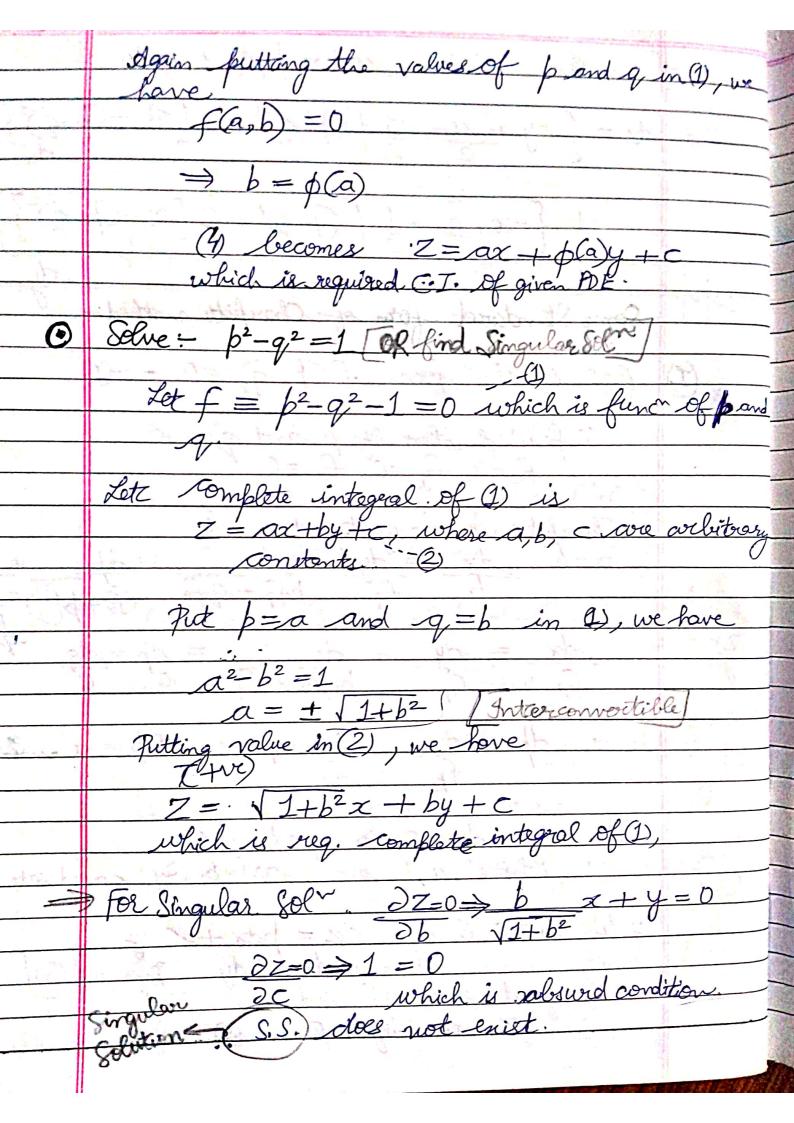
(p-x)+(q-y)=a-3 constant.

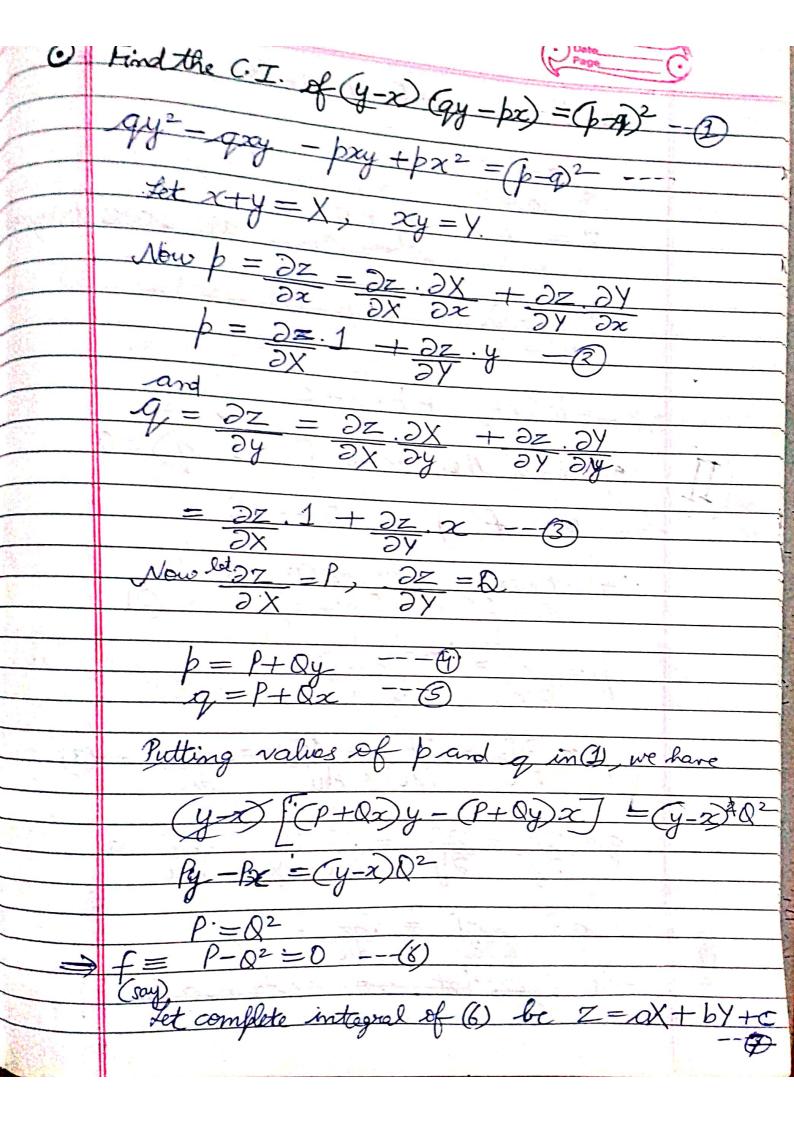


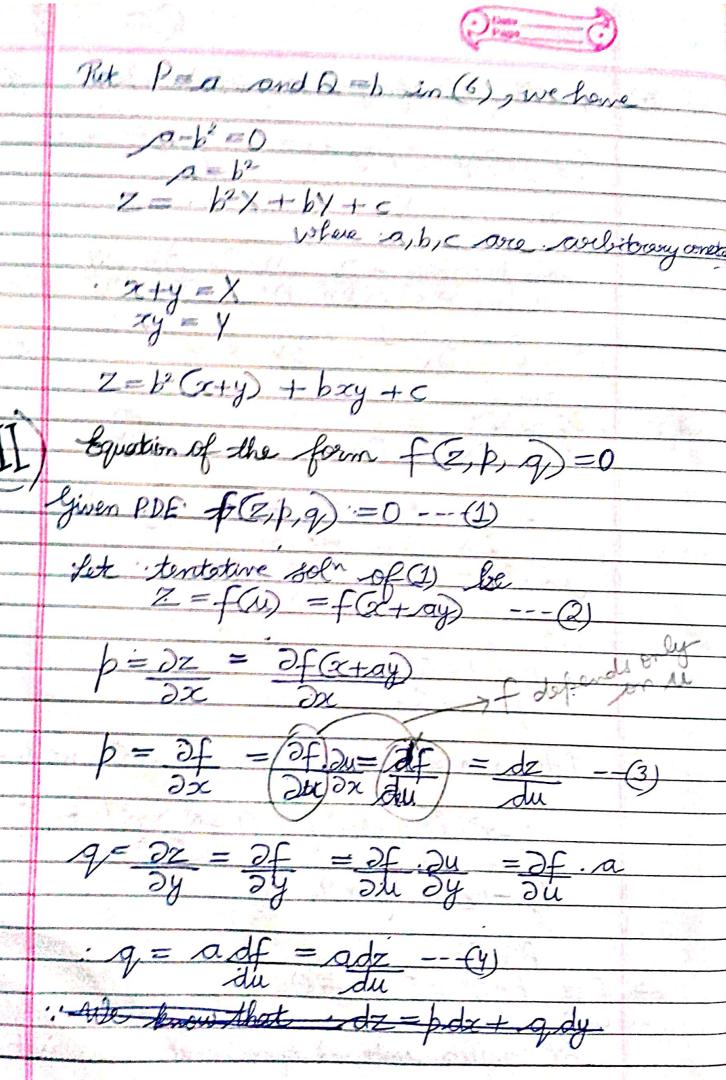
: We know that dz = pdz + qdy $dz = \left\{ x + a + \frac{1}{2} \left[2(x - y^2 - a^2)^{\frac{1}{2}} \right] dx + \frac{1}{2} \left[2(x - y^2 - a^2) \right] dx + \frac{1}{2} \left[2(x - y^2 - a^2) \right] dx + \frac{1}{2} \left[2(x - y^2 - a$ {y+a+1[26-y2-a2]42}dy $dz = x dx + y dy + q (dx + dy) + \frac{1}{2} \left[\frac{7}{2} (-y^2 - a^2)^2 \right]$ Integrating on both sides, (3x-dy) $2 = x^2 + y^2 + A(x+y) + (3x-dy)$ $z = \frac{x^2 + y^2 + A(x+y) + \cdots}{2 + 2(x+y) + \cdots}$ $\frac{1}{\sqrt{2}} \left[\frac{\chi - y}{2} \sqrt{(\chi - y)^2 - a^2} - \frac{a^2 \log \chi - y}{2} + \frac{1}{\sqrt{2}} \sqrt{2} \sqrt{(\chi - y)^2 - a^2} - \frac{a^2 \log \chi - y}{2} + \frac{1}{\sqrt{2}} \sqrt{2} \sqrt{2} \sqrt{2} \right]$ $2z = x^2 + y^2 + ax + ay + \frac{1}{\sqrt{2}} \left[(x-y) \cdot \sqrt{(x-y^2)^2} \right]$ 3) Solve: $f = 2(pq + py + qx) + (x^2 + y^2) = 0 - - 9$ $\frac{dp}{-(q+x)} = \frac{dq}{-(q+y)} = \frac{dx}{2(q+y)} = \frac{dy}{2(p+x)}$ $\frac{dp + dq + dx + dy}{xxy} = 0$ Integrating both sides -2 where a is arbitrary constant

2/2+2/y+2/2x+22+y2=0 2/9 + (y+p)2-p2 + (x+q)2-q2=0 (y+p2+ (x+q2-(p-2)=0 9+p2+ 5x+93= (p-2)2 Let y+p=P, x+q=Q, $P^2 + Q^2 = (p-q)^2 - - - (3)$ and eq. (2) becomes P+0 = 12 - - (9)Now $(P-Q)^2 = P^2 + Q^2 - 280$ $= P^2 + Q^2 - 2[P+Q)^2 - (P^2+Q^2)$ = (p-q)2-2[a2-(p-q)] $P-Q = \{ (p-p)^2 - 2 [\sigma^2 - (p-q)^2] \}^{\frac{4}{12}}$ Adding (9) and (5), we have $P = \frac{a}{2} + \frac{1}{2} \left[(p-q)^2 - 2 \left[\sigma^2 - (p-q)^2 \right]^2 \right]$ subtracting 9 and 3, we got $0 = \frac{a-1}{2} \left[(p-q)^2 - 2 \left[a^2 - (p-q)^2 \right] \right]^2$ p=-y+9+11 "1 [From []









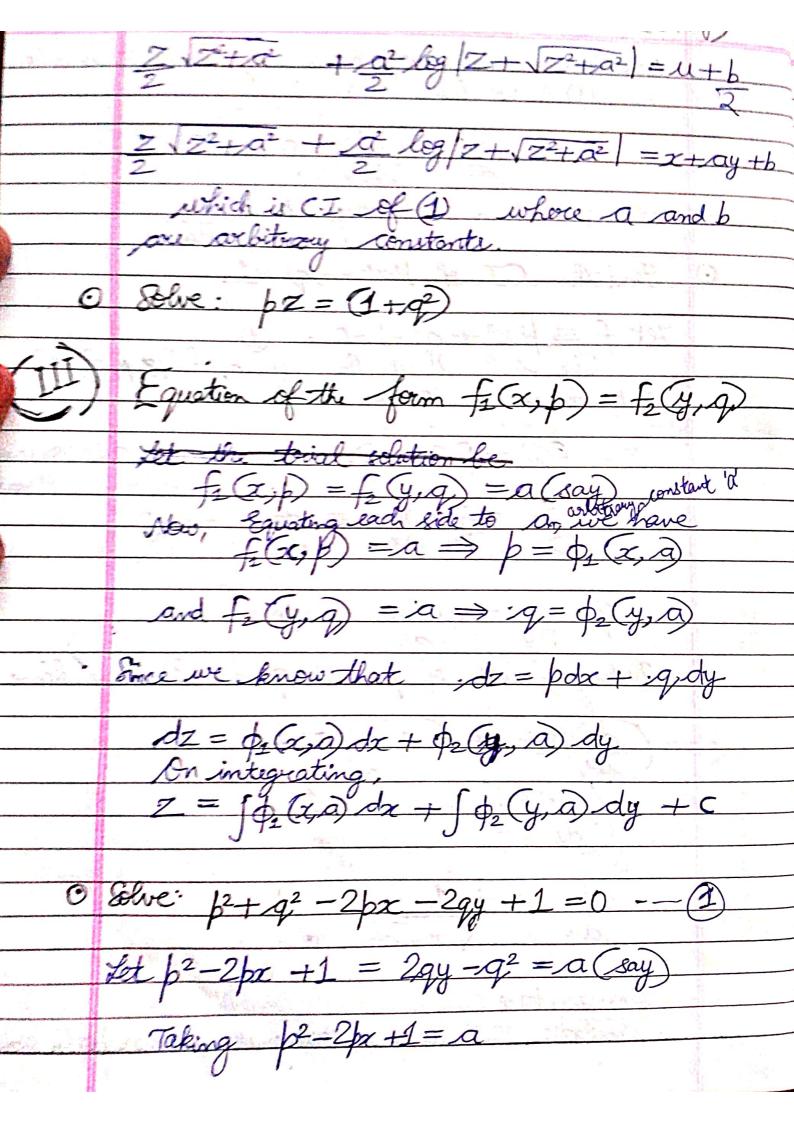
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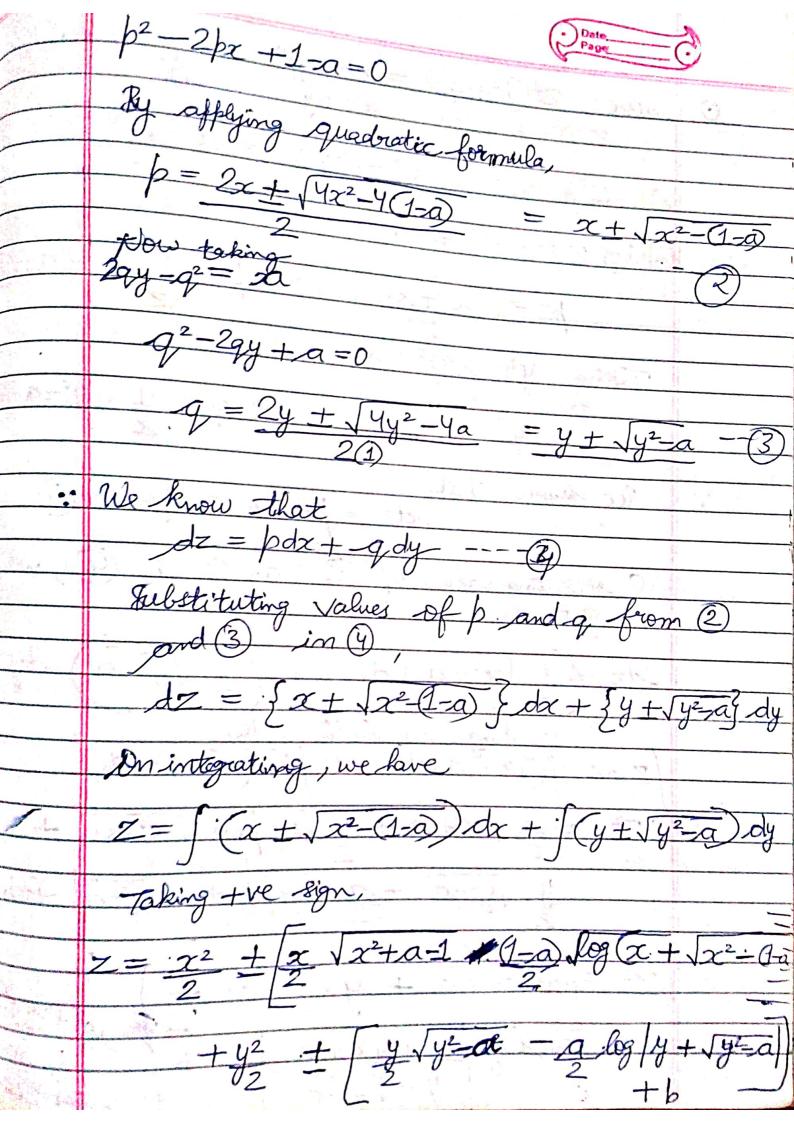
Substituting values of frank of from (2) and (3) f(Z, g, par)=0...(5) which is sordinary to in de O Find the CI if p22 -1- q2 =1 Fet ferstoking relation of (2) be (2) 100. $Z = f(x) = f(x + py) - 2 \quad \text{when} \quad p(x) = p(x)$ p= 22 = 32 24 = 34 (2) Tetting values of p and q in (1) just love

(dz) 2 + (adz) = 1

(dz) 2 [z²+p²] = 1

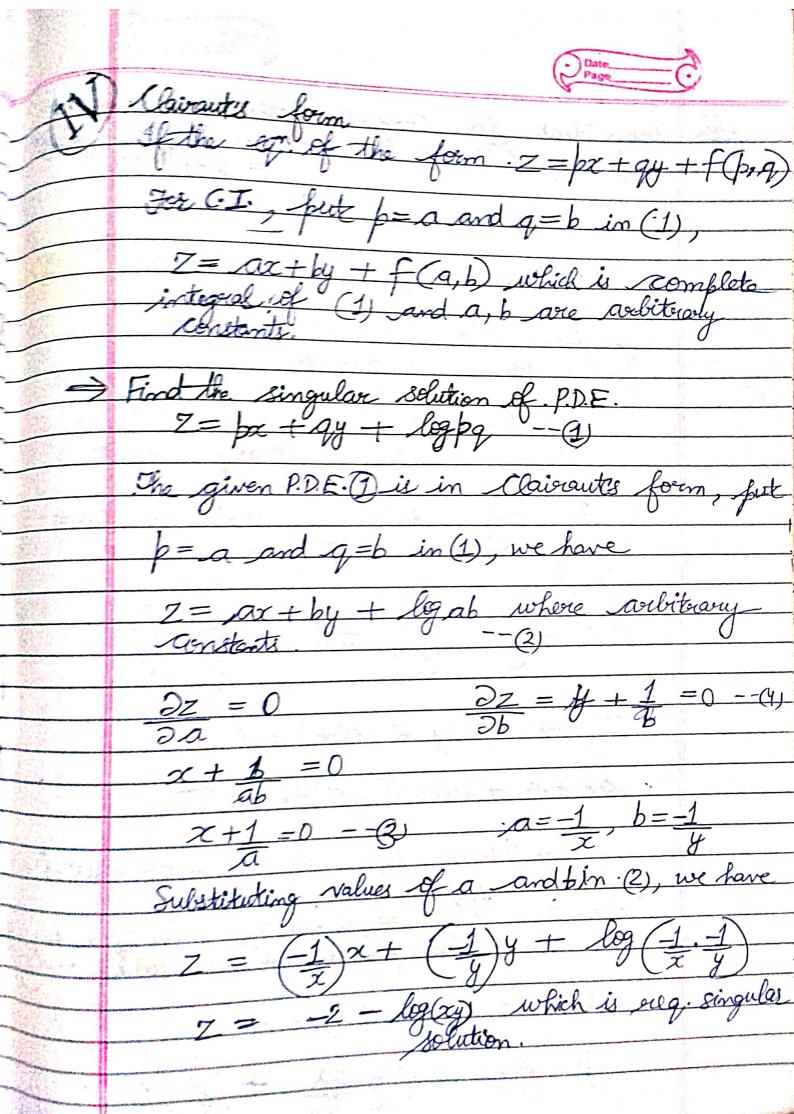
(dz) 2 [z²+p²] Marine Ma Taking the sign, THEdz = du Britigatary on both sides,

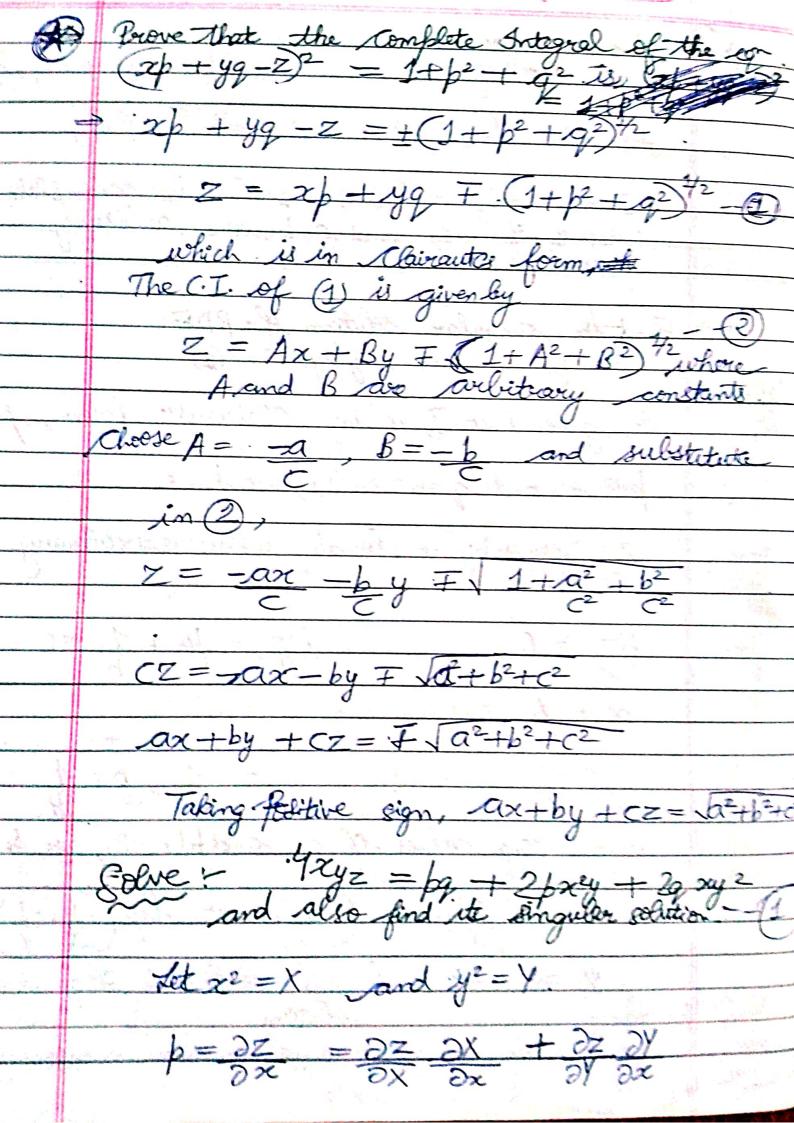




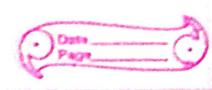
O Solve: $Z^2(p^2+q^2) = x^2+y^2$ $Z^2p^2 + Z^2q^2 = x^2+y^2 - -0$ Let $Z^2 p^2 - x^2 = y^2 - Z^2 q^2 = 0$ (sy) aking $Z^2\beta^2-\chi^2=0$ $b^{2} = x^{2} + a \Rightarrow b = 1 \sqrt{x^{2} + p - 2}$ z^{2} $z^{2} = x^{2} + a$ $z^{2} = a$ Taking y2-22g2=a $q^{2} = y^{2} - a \Rightarrow q = 1 \sqrt{y^{2} - a - 3}$ ||a| ||b| ||a| - ||a|| = 1:: We know that dz = pdx + q dy - QSubstituting values of Q and Q in Q we

get $\frac{dz}{z} = \left(\frac{1}{z}\sqrt{x^2+a}\right)dx + \left(\frac{1}{z}\sqrt{y^2-a}\right)dy$ Integrating on both sides. $\left(\frac{1}{z}\sqrt{x^2+a}\right)dx + \left(\sqrt{y^2-a}\right)dy$ $\frac{Z^{2}}{2} = x \sqrt{x^{2} + a} + a \log |x + \sqrt{x^{2} + a}| + \frac{1}{2}$ 4 Jy2-a -a log y + Jy2-a + b $7 = \left[x \sqrt{x^{2} + a} + y \sqrt{y^{2} - a} + \frac{1}{4} y \sqrt{x^{2} + a} \right] + \frac{1}{4} y \sqrt{x^{2} - a}$ $a \log \left[x + \sqrt{x^{2} + a} \right] + b + c$ $y + \sqrt{y^{2} - a}$ a constants





Pulling values of pard q in (1), we have 479/2 - pg (2x2) (2y2) + 2(2x2) + 2(2y 2z) 2y2 $Z = \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} + \frac{\partial z}{\partial y} \frac{\partial z}{\partial y} + \frac{\partial z}{\partial y} \frac{\partial z}{\partial y} - (2)$ Let $\frac{\partial z}{\partial x} = P$, $\frac{\partial z}{\partial y} = Q - 3$ raing 3 in D Z = PQ + PX + QY - G, which is in Clairants from whose GT of (3) is
Z = ab +aX +by where as b core Putting solutions of X and Y $Z = ab + ax^2 + by^2 - - - (5)$ $\frac{\text{For S.S.}}{\partial z} = b + \varkappa^2 = 0$ $\frac{\partial z}{\partial b} = a + y^2 = 0$



Substituting the values of a and b in (5), we get the ray S.J. $Z = -\frac{2y^2}{7}x^2y^2 - \frac{2y^2}{7} = -\frac{x^2y^2}{7}$ $Z + \frac{x^2y^2}{7} = 0$