Partial Differentiation

Partial differential coefficient :

Let $z = f(x, y)$ be function of two variables x and y, then partial derivative of z w.r.t x is defined as its ordinary differential coefficient w.r.t x obtained by treating other variable y as constant. It is denoted by $\frac{\partial z}{\partial x}$.

Thus $$\frac{\partial z}{\partial x} = \underset{\delta x \to 0}{Lt} \frac{f(x+\delta x, y) - f(x, y)}{\delta x} = f_x$$

Provided the limit exist.

Similarly the partial derivative of z w.r.t y is its ordinary differential coefficient w.r.t. y obtained by treating the other variable x as constant. It is denoted by $\frac{\partial z}{\partial y}$

$$\frac{\partial z}{\partial y} = \underset{\delta y \to 0}{Lt} \frac{f(x, y+\delta y) - f(x, y)}{\delta y} = f_y$$

Provided the limit exist.

Second order partial derivatives of f w.r.t. x & y are

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} \text{ (or } f_{xx}) \quad \text{and} \quad \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} \text{ (or } f_{yy})$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} \text{ (or } f_{yx}) \quad \text{and} \quad \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} \text{ (or } f_{xy})$$

If $u = f(x, y)$ possesses continuous partial derivatives, then

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

Q1 If $u = x^2 \tan^{-1}(y/x) - y^2 \tan^{-1}(x/y)$, then find $\frac{\partial^2 u}{\partial x \partial y}$

Sol. Differentiating partially w.r.t y (treating x as constant)

$$\frac{\partial u}{\partial y} = x^2 \cdot \frac{1}{1+(y/x)^2} \cdot \frac{1}{x} - 2y\tan^{-1}(x/y) + y^2 \cdot \frac{1}{1+(x/y)^2} \cdot \frac{x}{y^2}$$

$$= \frac{x^3}{x^2+y^2} + \frac{xy^2}{x^2+y^2} - 2y\tan^{-1}\left(\frac{x}{y}\right)$$

$$= x - 2y\tan^{-1}\left(\frac{x}{y}\right)$$

now differentiating partially w.r.t x (treating y as constant)

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = 1 - \frac{2y}{1+(x/y)^2} \cdot \frac{1}{y}$$

$$= 1 - \frac{2y^2}{x^2+y^2} = \frac{x^2-y^2}{x^2+y^2}$$

or $$\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}$$

Q2 If $u = (x^2+y^2+z^2)^{-1/2}$, Prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -u$$

Sol. Differentiating partially w.r.t x (treating y & z as constant)

$$\frac{\partial u}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} \cdot 2x = -x(x^2+y^2+z^2)^{-3/2}$$

$$\therefore \quad x\frac{\partial u}{\partial x} = -x^2(x^2+y^2+z^2)^{-3/2} \quad \text{---- (1)}$$

Similarly $$y\frac{\partial u}{\partial y} = -y^2(x^2+y^2+z^2)^{-3/2} \quad \text{---- (2)}$$

and $$z\frac{\partial u}{\partial z} = -z^2(x^2+y^2+z^2)^{-3/2} \quad \text{---- (3)}$$

Add. (1), (2) and (3), we have

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -(x^2+y^2+z^2)^{-1/2} = -u$$

Q3 If $u = \log(x^3+y^3+z^3-3xyz)$, then prove that

(i) $$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$

(ii) $$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

Sol. (a) $$\frac{\partial u}{\partial x} = \frac{3x^2-3yz}{x^3+y^3+z^3-3xyz} \quad \text{-- (1)}$$

also $$\frac{\partial u}{\partial y} = \frac{3y^2-3xz}{x^3+y^3+z^3-3xyz} \quad \text{-- (2)}$$

$$\frac{\partial u}{\partial z} = \frac{3z^2-3xy}{x^3+y^3+z^3-3xyz} \quad \text{-- (3)}$$

Adding (1), (2) and (3)

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3(x^2+y^2+z^2-xy-yz-zx)}{x^3+y^3+z^3-3xyz}$$

$$= \frac{3\cancel{(x^2+y^2+z^2-xy-yz-zx)}}{(x+y+z)\cancel{(x^2+y^2+z^2-xy-yz-zx)}}$$

$$= \frac{3}{x+y+z}$$

now $$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\right)$$

$$= \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\left(\frac{3}{x+y+z}\right) \quad \text{(from (i))}$$

$$= \frac{\partial}{\partial x}\left(\frac{3}{x+y+z}\right) + \frac{\partial}{\partial y}\left(\frac{3}{x+y+z}\right) + \frac{\partial}{\partial z}\left(\frac{3}{x+y+z}\right)$$

$$= \frac{-3}{(x+y+z)^2} - \frac{3}{(x+y+z)^2} - \frac{3}{(x+y+z)^2}$$

$$= \frac{-9}{(x+y+z)^2}$$

Q4 If $u = e^{xyz}$, then prove that :

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$

Sol. Differentiating partially w.r.t z, we have

$$\frac{\partial u}{\partial z} = e^{xyz}\, xy \quad \text{---} (1)$$

now differentiating (1) w.r.t y (treating x & z constant)

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial z}\right) = x\left[e^{xyz}\cdot 1 + y e^{xyz}(xz)\right]$$

$$\Rightarrow \frac{\partial^2 u}{\partial y \partial z} = e^{xyz}(x + x^2yz) \quad \text{--} (2)$$

differentiating (2) w.r.t x, we have

$$\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial y \partial z}\right) = e^{xyz}(1 + 2xyz) + e^{xyz}\, yz\,(x + x^2yz)$$

$$= e^{xyz}(1 + 3xyz + x^2y^2z^2)$$

Q5 If $z = f(x+ay) + \phi(x-ay)$, Prove that

$$\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$$

Sol. $$\frac{\partial z}{\partial x} = f'(x+ay) + \phi'(x-ay)$$

and $$\frac{\partial^2 z}{\partial x^2} = f''(x+ay) + \phi''(x-ay) \quad \cdot\cdot (1)$$

now differentiating partially w.r.t y

$$\frac{\partial z}{\partial y} = a f'(x+ay) - a\phi'(x-ay) = a\{f'(x+ay) - \phi'(x-ay)\}$$

and $$\frac{\partial^2 z}{\partial y^2} = a^2\left[f''(x+ay) + \phi''(x-ay)\right] = a^2\frac{\partial^2 z}{\partial x^2} \quad [\text{from } (1)]$$

Homogeneous function :- A function $f(x,y)$ of two variables x and y is said to be a homogeneous function of degree n, if the sum of indices of x and y in every term is same and it is equal to n

eg $f(x,y) = a_0 x^n + a_1 x^{n-1} y + a_2 x^{n-2} y^2 + \cdots + a_n y^n$

is homogeneous function of degree n, where $a_0, a_1, \ldots a_n$ are constant.

→ function $f(x,y)$ is homogeneous function of degree n in x & y if it can be written as

(i) $f(\lambda x, \lambda y) = \lambda^n f(x,y)$

(ii) $f(x,y) = x^n \phi(y/x)$

(iii) $f(x,y) = y^n \psi(x/y)$

eg. (1) $f(x,y) = x^2 + xy$

$x \to \lambda x, \quad y \to \lambda y$

$f(\lambda x, \lambda y) = (\lambda x)^2 + (\lambda x)(\lambda y)$

$= \lambda^2 x^2 + \lambda^2 xy = \lambda^2 (x^2 + xy)$

$= \lambda^2 f(x,y)$ (form (i))

So $f(x,y)$ is homogeneous function with degree 2

(2) $\tan^{-1}(y/x)$ is homogeneous function with degree zero

(3) $\dfrac{x^2+y}{x+y^2}$ is not homogeneous function of x & y.

Euler's theorem on homogeneous functions

If $f(x, y)$ be homogenous function of x & y of degree n then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$

proof Since $f(x, y)$ is homogenous function of degree n so it can be written as

$$f(x, y) = x^n \phi(y/x) \quad \text{---(1)}$$

Differentiating Partially w.r.t x

$$\frac{\partial f}{\partial x} = x^n \phi'(y/x)\left(-\frac{y}{x^2}\right) + n x^{n-1}\phi(y/x) \quad \text{--(2)}$$

mult. by x, we have

$$x\frac{\partial f}{\partial x} = -x^{n-1} y\, \phi'(y/x) + n x^n \phi(y/x) \quad \text{--(3)}$$

Differentiating partially (1) wrt y

$$\frac{\partial f}{\partial y} = x^n \phi'(y/x)\cdot\frac{1}{x} = x^{n-1}\phi'(y/x) \quad \text{--(4)}$$

mult. by y

$$y\frac{\partial f}{\partial y} = y x^{n-1} \phi'(y/x) \quad \text{-(5)}$$

adding (3) & (5)

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = n x^n \phi(y/x) = nf \quad (\text{using } (1))$$

Generalisation : If $f(x_1, x_2, \ldots x_m)$ be homogeneous function of m variables of degree n, then

$$x_1\frac{\partial f}{\partial x_1} + x_2\frac{\partial f}{\partial x_2} + \cdots + x_m\frac{\partial f}{\partial x_m} = nf$$

Q1 If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$

Soln $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$\Rightarrow \cos u = \frac{x+y}{\sqrt{x}+\sqrt{y}} = f(x,y)$

$f(x,y)$ is homogeneous function of x & y of degree $\frac{1}{2}$ (check) i.e. $n = \frac{1}{2}$

Using Euler's theorem

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf = \frac{f}{2}$$

$$\Rightarrow x\frac{\partial}{\partial x}(\cos u) + y\frac{\partial}{\partial y}(\cos u) = \frac{\cos u}{2}$$

$$\Rightarrow -x\sin u\frac{\partial u}{\partial x} - y\sin u\frac{\partial u}{\partial y} = \frac{\cos u}{2}$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{\cot u}{2}$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{\cot u}{2} = 0$$ Proved

Q2 If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}(y/x)$ then prove

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

Soln $u(\lambda x, \lambda y) = \sin^{-1}\left(\frac{\lambda x}{\lambda y}\right) + \tan^{-1}\left(\frac{\lambda y}{\lambda x}\right) = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}(y/x)$

$= u(x,y)$

$= \lambda^0 u(x,y)$

so u is homo. funct^n of x & y of degree zero. $(n=0)$

Using Euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu = 0u = 0$$ Proved

Q3 If $u = \log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$

Soln $u = \log\left(\frac{x^4+y^4}{x+y}\right)$ is not homo. function

$e^u = \frac{x^4+y^4}{x+y} = f(x,y)$, f is homo function of x & y of degree 3

Using Euler's th.

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf = 3f$$

$$\Rightarrow x\frac{\partial}{\partial x}(e^u) + y\frac{\partial}{\partial y}(e^u) = 3e^u$$

$$\Rightarrow xe^u\frac{\partial u}{\partial x} + ye^u\frac{\partial u}{\partial y} = 3e^u \Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$$ HP

Q4 If $u = \tan^{-1}\left(x^2 + \frac{y^3}{x} + y^2\right)$ then prove $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

Soln $\tan u = x^2 + \frac{y^3}{x} + y^2 = f(x,y)$ say.

$$f(\lambda x, \lambda y) = (\lambda x)^2 + \frac{(\lambda y)^3}{(\lambda x)} + (\lambda y)^2 = \lambda^2 x^2 + \lambda^2\frac{y^3}{x} + \lambda^2 y^2$$
$$= \lambda^2\left(x^2 + \frac{y^3}{x} + y^2\right) = \lambda^2 f(x,y)$$

So $f(x,y)$ is homo. functⁿ of x & y of degree 2.

using Euler's Th. $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf = 2f$

$$\Rightarrow x\frac{\partial}{\partial x}(\tan u) + y\frac{\partial}{\partial y}(\tan u) = 2\tan u$$

$$\Rightarrow x\sec^2 u\frac{\partial u}{\partial x} + y\sec^2 u\frac{\partial u}{\partial y} = 2\tan u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{2\tan u}{\sec^2 u} = \sin 2u$$ HP

If u is a homogeneous function of degree n, then

i) $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = (n-1)\frac{\partial u}{\partial x}$

ii) $x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = (n-1)\frac{\partial u}{\partial y}$

iii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$

Proof (i) Since u is homogeneous function of x & y of degree n, from Euler's th., we have

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu \quad \text{--- (1)}$$

Differentiating both sides of (1) partially w.r.t x

$$x\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + y\frac{\partial^2 u}{\partial x \partial y} = n\frac{\partial u}{\partial x}$$

$$\Rightarrow x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = (n-1)\frac{\partial u}{\partial x} \quad \text{--- (2)}$$

again diff. both sides of (1) partially w.r.t y, we get

$$x\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial u}{\partial y} + y\frac{\partial^2 u}{\partial y^2} = n\frac{\partial u}{\partial y}$$

$$\Rightarrow x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = (n-1)\frac{\partial u}{\partial y} \quad \text{--- (3)}$$

$\left(\text{Since } \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}\right)$

mult. (2) by x & (3) by y and adding, we get

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (n-1)\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right]$$

$$= n(n-1)u \quad [\text{Using (1)}]$$

Q1 If $u = x\phi(y/x) + \psi(y/x)$, Prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$$

Solution $u = x\phi(y/x) + \psi(y/x)$

Let $x\phi(y/x) = v$ and $\psi(y/x) = \omega$ then $u = v + \omega$ — (1)

clearly v & ω are homogeneous function of degree 1 & 0

By Euler's theorem

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = v \quad \text{— (2)}$$

$$x\frac{\partial \omega}{\partial x} + y\frac{\partial \omega}{\partial y} = 0 \quad \text{— (3)}$$

adding (2) & (3)

$$x\frac{\partial}{\partial x}(v+\omega) + y\frac{\partial}{\partial y}(v+\omega) = v$$

$$\Rightarrow \quad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = v \quad (4) \text{(from (1))}$$

Now diff. both sides of (4) partially w.r.t x

$$x\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + y\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial v}{\partial x}$$

$$\Rightarrow \quad x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial x} \quad \text{— (5)}$$

again diff. (4) partially w.r.t y

$$x\frac{\partial^2 u}{\partial y \partial x} + y\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}$$

$$\Rightarrow \quad x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \quad \text{— (6)}$$

mult. (5) by x & (6) by y and adding

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = x\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\right) + y\left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\right)$$

$$= \left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right) - \left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)$$

[by (2) & (4)]

$$= v - v$$

$$= 0 \text{ proved}$$

Q2 If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1-4\sin^2 u)\sin 2u$$

Soln Given $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$

$$\Rightarrow \tan u = \frac{x^3+y^3}{x-y} = f(x,y)$$

f is homo. functn of x & y of degree 2

Using Euler's theorem

$$x\frac{\partial}{\partial x}f + y\frac{\partial}{\partial y}f = nf = 2f$$

$$\Rightarrow x\frac{\partial}{\partial x}(\tan u) + y\frac{\partial}{\partial y}(\tan u) = 2\tan u$$

$$\Rightarrow x\sec^2 u\frac{\partial u}{\partial x} + y\sec^2 u\frac{\partial u}{\partial y} = 2\tan u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u \quad \text{--- (1)}$$

diff. both sides (1) partially. wrt x

$$x\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + \frac{y\,\partial^2 u}{\partial x \partial y} = 2\cos 2u\frac{\partial u}{\partial x}$$

$$\Rightarrow x\frac{\partial^2 u}{\partial x^2} + \frac{y\,\partial^2 u}{\partial x \partial y} = (2\cos 2u - 1)\frac{\partial u}{\partial x} \quad \text{--- (2)}$$

diff. both sides (1) partially wrt y

$$x\frac{\partial^2 u}{\partial y \partial x} + \frac{y\,\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = 2\cos 2u\frac{\partial u}{\partial y}$$

$$\Rightarrow x\frac{\partial^2 u}{\partial x \partial y} + \frac{y\,\partial^2 u}{\partial y^2} = (2\cos 2u - 1)\frac{\partial u}{\partial y} \quad \text{--- (3)}$$

mult. (2) by x & (3) by y & then adding

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (2\cos 2u - 1)\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)$$

$$= [2(1-2\sin^2 u) - 1]\sin 2u \text{ (from (1))}$$

$$= (1-4\sin^2 u)\sin 2u$$

Proof

Problem set-03 (Partial Differentiation)

Q1 If $u = \sin^{-1}\left(\frac{x}{y}\right)$, verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

Q2 If $z = \log_e(x^2+y^2) + \tan^{-1}(y/x)$, then Prove that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

Q3 If $z = \frac{x^2+y^2}{x+y}$, then Prove that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

Q4 If $u = e^x(x\cos y - y\sin y)$, then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Q5 If $\theta = t^n e^{-r^2/4t}$, find value of n such that

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$$