Unit-II

Page
Date

Completely Randomized Design (CRD) :-

The CRD is the simplest of all designs based on the principles of randomisation and replication. In this design the treatments are allocated at random to the experimental units over the entire experimental material.

Let us suppose we have v treatments, the i^{th} treatment being replicated r_i times $i = 1, 2, \ldots, v$. Then the whole experimental material is divided into $n = \Sigma r_i$, experimental units and the treatments are distributed completely at random over the units subject to the condition that the i^{th} treatment occurs r_i times.

In particular case if

$$r_i = r \quad \forall \; i = 1, 2, \ldots v$$

i.e. if each treatment is repeated an equal no of times r, then $n = rv$ and randomisation gives every group of r units an equal chance of receiving the treatments.

Statistical Analysis of C.R.D :- Statistical analysis of CRD is analogous to the ANOVA for a one-way classified data, the linear model becomes

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij} \quad ; \quad \left(\begin{array}{l} i = 1, 2, \ldots v \\ j = 1, 2, \ldots r_i \end{array} \right)$$

where y_{ij} is the yield or response from the j^{th} unit, receiving the i^{th} treatments, μ is the general mean effect, τ_i is the effect due to the i^{th} treatment and e_{ij} is error effect due to chance s.t. e_{ij} is identically and independently distributed (i.i.d.) $N(0, \sigma_e^2)$. then

$n = \Sigma r_i$ is the total no. of experimental units. If we write

$\sum_i \sum_j y_{ij} = y_{..} = G$ = Grand total of all the n obsn.

$\sum_j^{r_i} y_{ij} = y_{i.} = T_i$ = Total response of the units receiving the i^{th} treatments.

then as in ANOVA (one way classified data)

$$\sum_i \sum_j (y_{ij} - \bar{y}_{..})^2 = \Sigma\Sigma (y_{ij} - \bar{y}_{i.})^2 + \Sigma r_i (\bar{y}_{i.} - \bar{y}_{..})^2$$

i.e. T.S.S. = S.S.E. + S.S.T.

where T.S.S., S.S.T and S.S.E. are the total sum of squares, sum of squares due to treatments (betn treatment S.S.) and sum of square due to error (i.e. within treatment S.S) given respectively the

$$T.S.S. = \sum_i \sum_j (y_{ij} - \bar{y}_{..})^2$$

$$S.S.T. = \sum_i r_i (\bar{y}_{i.} - \bar{y}_{..})^2 = S_T^2 \text{ (say)}$$

$$S.S.E. = \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2 = S_E^2 \text{ (say)}$$

Page
Date

ANOVA TABLE FOR CRD

Source of variation	d.f.	S.S.	M.S.S.	Variance ratio (F)
Treatments	$v-1$	S_T^2	$s_T^2 = \frac{S_T^2}{v-1}$	$F_T = \frac{s_T^2}{s_E^2}$
Error	$n-v$	S_E^2	$s_E^2 = \frac{S_E^2}{n-v}$	
Total	$n-1$	$S_T^2+S_E^2$		

under the null hypothesis $H_0 : \tau_1 = \tau_2 = \cdots = \tau_v$ against the alternative that all τ's are not equal, the statistic

$$F_T = \frac{s_T^2}{s_E^2} \sim F(v-1, n-v)$$

i.e. F_T follows F (central) distribution with $(v-1, n-v)$ d.f.

If $F_T > F_{\alpha}(v-1, n-v)$ then H_0 is refuted at $\alpha\%$ level of significance and we conclude that treatment differ significantly. If $F_T < F_{\alpha}(v-1, n-v)$ H_0 may be accepted i.e. the data do not differ significantly and we conclude all treatments alike.

Remark :- The following formulae for the calculation of various S.S. are much convenient to use from practical point of view

$$T.S.S. = \sum_i\sum_j (y_{ij} - \bar{y}_{..})^2 = \sum_i\sum_j y_{ij}^2 - n\bar{y}_{..}^2$$

$$= \sum\sum y_{ij}^2 - n\left(\frac{\sum\sum y_{ij}}{n}\right)^2$$

$$= \sum\sum y_{ij}^2 - \frac{G^2}{n} \quad ; \quad n = \sum r_i$$

T.S.S. = Raw S.S. − correction factor

= R.S.S. − C.F.

$$S.S.T. = \sum r_i (\bar{y}_{i.} - \bar{y}_{..})^2$$

$$= \sum r_i \bar{y}_{i.}^2 - n\bar{y}_{..}^2$$

$$= \sum_{i=1}^{v} \left(\frac{T_i^2}{r_i}\right) - C.F.$$

S.S.E. = T.S.S. − S.S.T.

$$= \sum_i\sum_j y_{ij}^2 - \sum\left(\frac{T_i^2}{r_i}\right)$$

$$= \sum_i\sum_j y_{ij}^2 - \sum_i r_i \bar{y}_{i.}^2$$

Standard error of C.R.D.

Standard error of the difference between any two treatment means is given by

$$s_E\sqrt{\frac{2}{r}}$$

$\therefore$ Critical difference (C.D.) $= \sqrt{\frac{2}{r}s_E^2} \times t_{0.05}$ error d.f.

critical differnce is measure to find that when Ho is rejected and we want to know by which pair of treatment, means differ significantly i.e. the least difference betⁿ any two means to be significant.

Merits & Demerits :—

Merits :— ① flexible ② easy to analysis ③ all experimental material is used. ④ Provides maxᵐ no. of degrees of freedom that increase the precision or sensitivity

Demerits :— ① Inherently less informative ② no restriction on randomisation.

Application :— CRD is mostly applicable in laboratory techniques. It is widely used in chemical & biological experiments, Methodology studies of cookery and physics and chemistry experiments.