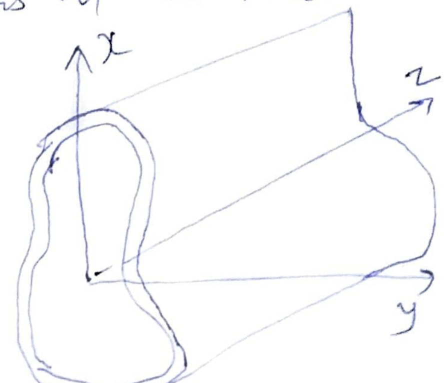
UNIT - III

Guided Waves

Wave Guides :- Consider E.M. waves confined to the interior of a hollow pipe or wave guide. We assume the wave guide is a perfect conductor, so that $E = 0$ and $B = 0$ inside the material itself, and hence the boundary conditions at the inner wall are

$$\left.\begin{array}{l} (i)\ E^{\parallel} = 0 \\ (ii)\ B^{\perp} = 0 \end{array}\right\} \quad \text{--- (1)}$$



in a perfect conductor $E = 0$
and hence Faraday's law $\frac{\partial B}{\partial t} = 0$;
assuming magnetic field started out zero then, it will remain so

Free charges and currents will be induced on the surface in such a way as to enforce these constraints.

We are interested in monochromatic waves that propagate down the tube, so E and B have the general form

$$\left.\begin{array}{l} (i)\ \tilde{E}(x,y,z,t) = \tilde{E}_0(x,y)\, e^{i(kz-\omega t)} \\ (ii)\ \tilde{B}(x,y,z,t) = \tilde{B}_0(x,y)\, e^{i(kz-\omega t)} \end{array}\right\} \quad \text{--- (2)}$$

The electric and magnetic fields must, of course, satisfy Maxwell's eq^{n}s in the interior of the wave guide:

$$\left.\begin{array}{ll} (i)\ \nabla\cdot E = 0 & (iii)\ \nabla\times E = -\frac{\partial B}{\partial t} \\ (ii)\ \nabla\cdot B = 0 & (iv)\ \nabla\times B = \frac{1}{c^2}\frac{\partial E}{\partial t} \end{array}\right\} \quad \text{--- (3)}$$

The problem is to find functions $\tilde{E}_0$ and $\tilde{B}_0$ such that the fields (2) obey the differential eq^{n}s (3), subject to boundary conditions (1)

Confined waves are not, in general, transverse; in order to fit the boundary conditions we shall have to include longitudinal components

$$\tilde{E}_0 = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}, \quad \tilde{B}_0 = B_x\hat{x} + B_y\hat{y} + B_z\hat{z} \quad \text{--- (4)}$$

where each of the components is a function of x, y. Putting this into Maxwell's eq^{n}s (iii) & (iv), we obtain

$$\left.\begin{aligned}
&(i)\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z \qquad &&(iv)\ \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -\frac{i\omega}{c^2}E_z\\
&(ii)\ \frac{\partial E_z}{\partial y} - ikE_y = i\omega B_x \qquad &&(v)\ \frac{\partial B_z}{\partial y} - ikB_y = \frac{i\omega}{c^2}E_x\\
&(iii)\ ikE_x - \frac{\partial E_z}{\partial x} = i\omega B_y \qquad &&(vi)\ ikB_x - \frac{\partial B_z}{\partial x} = -\frac{i\omega}{c^2}E_y
\end{aligned}\right\} \quad \text{--- (5)}$$

Eqs (ii), (iii), (v) and (vi) can be solved for E_x, E_y, B_x, B_y

$$\left.\begin{aligned}
&(i)\ E_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right)\\
&(ii)\ E_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)\\
&(iii)\ B_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)\\
&(iv)\ B_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)
\end{aligned}\right\} \quad \text{--- (6)}$$

It suffices to determine the longitudinal components E_z and B_z; if we knew those, we could calculate all the others, just by differentiating.

Inserting eq^n (6) into the remaining Maxwell's eq^n yields uncoupled eq^n for E & B

$$\left.\begin{aligned}
&(i)\quad \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (\omega/c)^2 - k^2\right] E_z = 0 \\
&(ii)\quad \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (\omega/c)^2 - k^2\right] B_z = 0
\end{aligned}\right\} \quad \text{---(7)}$$

if $E_z = 0$ we call these TE ("Transverse electric") waves; if $B_z = 0$ they are called TM ("Transverse Magnetic") waves; If Both $E_z = 0$, $B_z = 0$, we call the TEM waves.

TEM waves can not occur in a hollow wave guide

Proof: if $E_z = 0$, Gauss's Law says (eq^n 3(i))

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = 0$$

and if $B_z = 0$, Faraday's Law says (eq^n 3(iii))

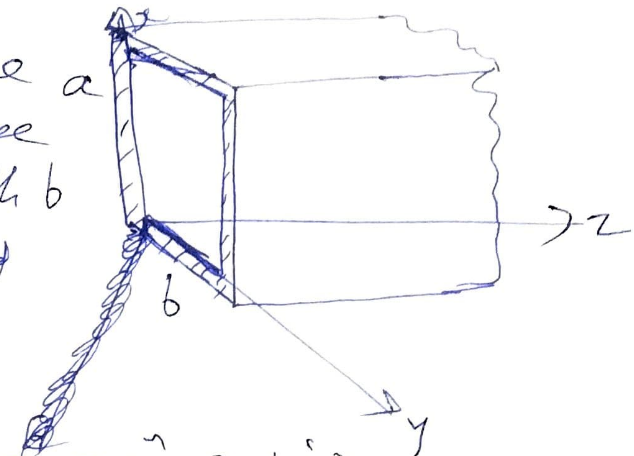
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0.$$

the vector $\tilde{E}_0$ in eq^n (4) has zero divergence & zero curl. It can therefore be written as the gradient of a scalar potential that satisfies Laplace's eq^n.

But the boundary condition on E (eq^n(1)) requires that the surface be an equipotential, and since Laplace's eq^n admits no local maxima or minima, this means that the potential is constant throughout, and hence the electric field is zero — no wave at all

TE Waves in a Rectangular Wave Guide

We have a wave guide of rectangular shape with height a and width b and we are interested of TE waves.



The problem is to solve eq^n 7 (ii) subject to boundary conditions (1)(ii)

Let $B_z(x,y) = X(x)\,Y(y)$ [separation of variables]

So that

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} + \left[(\omega/c)^2 - k^2\right]XY = 0$$

Divide by XY and note that the x- and y- dependent terms be constant:

$$\text{(i)}\ \frac{1}{X}\frac{d^2X}{dx^2} = -k_x^2 \qquad \text{(ii)}\ \frac{1}{Y}\frac{d^2Y}{dy^2} = -k_y^2 \quad \text{---}(8)$$

with

$$-k_x^2 - k_y^2 + (\omega/c)^2 - k^2 = 0 \quad \text{---}(9)$$

The general sol^n to eq^n 8(i) is

$$X(x) = A\sin(k_x x) + B\cos(k_x x)$$

But the boundary conditions require that B_x- and hence also (eq^n 6iii) $\frac{dX}{dx}$ - vanishes at $x=0$ and $x=a$, so $A=0$, and

$$k_x = m\pi/a, \quad (m=0,1,2\ldots) \quad \text{---}(10)$$

The same goes for Y, with

$$k_y = n\pi/b, \quad (n=0,1,2\ldots), \quad \text{---}(11)$$

and we conclude that

$$B_z = B_0\cos(m\pi x/a)\cos(n\pi y/b) \quad \text{---}(12)$$

Soln eqn (12) is called $\underline{TE}_{mn}$ mode.

first index is associated with ~~larger~~ larger

dimension, so we assume $a \geq b$.

at least one of the indice must be non-zero.

The wave number (k) is obtained by putting eqn (10) & (11) in eqn (9)

$$k = \sqrt{\left(\frac{\omega}{c}\right)^2 - \pi^2\left[\left(m/a\right)^2 + \left(\frac{n}{b}\right)^2\right]} \quad \text{---(13)}$$

if

$$\omega < c\pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(n/b\right)^2} \equiv \omega_{mn} \quad \text{---(14)}$$

The wave is imaginary, and instead of a ~~traveling~~ traveling wave we have exponentially attenuated fields. (Eqn (2))

$\omega_{mn} \longrightarrow$ called the cutoff frequency for the mode.

The lowest cutoff frequency for a given wave guide occurs for mode TE_{10}:

$$\omega_{10} = c\pi/a \quad \text{---(15)}$$

Frequencies less than this will not propagate at all

$$k = \frac{1}{c}\sqrt{\omega^2 - \omega_{mn}^2} \quad \text{---(16)}$$

The wave velocity is

$$v = \frac{\omega}{k} = \frac{c}{\sqrt{1-\left(\omega_{mn}/\omega\right)^2}} \quad \text{---(17)}$$

which is greater than c, However, the energy carried by the wave travels at the group velocity

$$v_g = \frac{1}{dk/d\omega} = c\sqrt{1-(\omega_{mn}/\omega)^2} < c \quad \text{--- (13)}$$

Resonant Cavities :- Electromagnetic Cavities

Purpose of waveguide → To transmit electromagnetic energy efficiently from one point to another.

Resonator → An energy storage device → equivalent to a resonant circuit element.

At high frequencies (100 MHz and above) the RLC circuit elements are insufficient when used as resonators → because the dimensions of the circuits are comparable with the operating wavelength → unwanted radiation takes place

At high frequencies the RLC circuit resonators → replaced by electromagnetic cavity resonators

↓ used in klystron tubes, bandpass filters and wave meters.

Microwave Oven → a power supply, a waveguide feed and an oven cavity.