UNIT IV

Dipole Radiation

Radiation? → Source is some arrangement of charge.

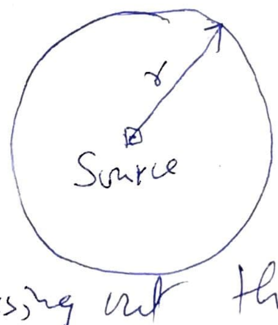
Charge at rest, steady current → No electromagnetic wave

Accelerating charge and changing current

How such configurations of charge produce → electromagnetic waves → i.e. How they radiate.

Signature of radiation :- Irreversible flow of energy away from the source.

We assume the source is localized near the origin.



Total power passing out through this surface is the integral of the Poynting vector:

$$P(r) = \oint S \cdot da = \frac{1}{\mu_0} \oint (E \times B) \cdot da \quad \text{---(1)}$$

The power radiated

$$P_{rad} \equiv \lim_{r \to \infty} P(r)$$

→ energy (per unit time)

i.e. transported out to infinity, and never comes back area of Sphere $\rightarrow 4\pi r^2$

Poynting vector must decrease (at large r) no faster than $\frac{1}{r^2}$

[if it went like $\frac{1}{r^3}$, $P(r) \rightarrow \frac{1}{r}$, $P_{rad} \rightarrow$ zero]

According to Coulomb's Law

Electrostatic fields fall off like $\frac{1}{r^2}$

Biot-Savart

Magnetostatic fields go like $\frac{1}{r^2}$

$S \sim \frac{1}{r^4}$ for static configurations

$\longrightarrow$ Static source do not radiate.

Jefimenko's eq^n $\longrightarrow$ time dependent fields include terms ($\dot{\rho}$ and $\dot{J}$) that go like $\frac{1}{r}$;

$\hookrightarrow$ these terms are responsible for electromagnetic radiation.

Study of radiation $\rightarrow$ picking out parts of E and B that go like $\frac{1}{r}$ at large distances from the source, constructing from them the $\frac{1}{r^2}$ term in S, integrating over a large spherical surface & taking limit as $r \longrightarrow \infty$

$$E(r,t) = \frac{1}{4\pi\varepsilon_0} \int \left[\frac{\rho(r', t_r)}{\mathscr{r}^2} \hat{\mathscr{r}} + \frac{\dot{\rho}(r', t_r)}{c\mathscr{r}} \hat{\mathscr{r}} - \frac{\dot{J}(r', t_r)}{c^2 \mathscr{r}} \right] d\tau'$$

$$B(r,t) = \frac{\mu_0}{4\pi} \int \left[\frac{J(r', t_r)}{\mathscr{r}^2} + \frac{\dot{J}(r', t_r)}{c\mathscr{r}} \right] \times \hat{\mathscr{r}} \, d\tau'$$

Continuous distribution

Retarded Potentials

$E = -\nabla V - \frac{\partial A}{\partial t}$ | $\nabla^2 - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \equiv \Box^2$ (d'Alembertian) --- (1)

$B = \nabla \times A$

Lorentz condition in the Maxwell's eq^n's | $\left.\begin{array}{l} \Box^2 V = -\frac{1}{\varepsilon_0}\rho \\ \Box^2 A = -\mu_0 J \end{array}\right\}$ --- (2)

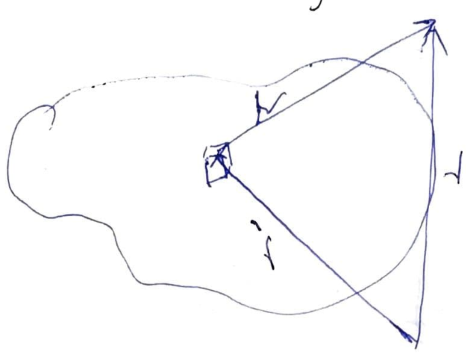
In the static case, eq^n (2) reduce to Poisson's eq^n

$$\nabla^2 V = -\frac{1}{\varepsilon_0}\rho \quad , \quad \nabla^2 A = -\mu_0 J$$

with the familiar sol^n

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')}{\mathscr{r}} d\tau' , \quad A(r) = \frac{\mu_0}{4\pi}\int \frac{J(r')}{\mathscr{r}} d\tau'$$

--- (3)



$\mathscr{r} \rightarrow$ distance from the source point r' to the field point r.

Now, electromagnetic "news" travels at the speed of light.

In nonstatic case $\rightarrow$ Status of the source right now does not matter, but rather its condition at some earlier time t_r (called the retarded time) when the "message" left.

Message must travel a distance $\mathscr{r}$, the delay is $\mathscr{r}/c$:

$$\boxed{t_r = t - \frac{\mathscr{r}}{c}} \quad \text{--- (4)}$$

Generalization of eq^n (3) for nonstatic sources is therefore

$$V(r,t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r', t_r)}{\mathscr{r}} d\tau' , \quad A(r,t) = \frac{\mu_0}{4\pi}\int \frac{J(r', t_r)}{\mathscr{r}} d\tau$$

--- (5)

$\rho(r', t_r) \rightarrow$ charge density that prevailed at point r' at the retarded time t_r.

Integrals evaluated at retarded time $\rightarrow$ these are called "retarded potentials".

Retarded potentials $\rightarrow$ reduce to eq^n (3) in the static case. $\rightarrow$ ρ & J are independent of time.

We did not derive formula for V & A
↳ e.m. news travels at the speed of light

To prove them $\rightarrow$ they satisfy inhomogeneous wave eq^n (2) and meet the Lorentz condition

$$\nabla \cdot A = -\mu_0 \varepsilon_0 \frac{\partial V}{\partial t}$$

If same argument applied to fields, we get entirely wrong answer

$$E(r,t) \neq \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r', t_r)}{\mathscr{r}^2} \hat{\mathscr{r}} \, d\tau'$$

$$B(r,t) \neq \frac{\mu_0}{4\pi} \int \frac{J(r', t_r) \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$$

Let's check, that the retarded scalar potential satisfies eq^n (2) $\rightarrow$ same argument will serve for vector potential.

Calculation of Laplacian of $V(r,t)$ $\rightarrow$ the integrand in eq^n (5) depends on r in two places: explicitly, in the denominator ($\mathscr{r} = |r - r'|$) and implicitly through $t_r = t - \frac{\mathscr{r}}{c}$, in the numerator.

Thus

$$\nabla V = \frac{1}{4\pi\varepsilon_0}\int\left[(\nabla\rho)\frac{1}{\imath} + \rho\,\nabla\left(\frac{1}{\imath}\right)\right]d\tau' \quad -(6)$$

and

$$\nabla\rho = \dot\rho\,\nabla t_r = -\frac{1}{c}\dot\rho\,\nabla\imath \quad --(7)$$

$\left[\frac{\partial}{\partial t_r} = \frac{\partial}{\partial t}\right.$, since $t_r = t - \frac{\imath}{c}$ and $\imath$ is independent of $t\left.\right]$

Now $\nabla\imath = \hat{\imath}$ and $\nabla\left(\frac{1}{\imath}\right) = -\frac{\hat{\imath}}{\imath^2}$

$$\nabla V = \frac{1}{4\pi\varepsilon_0}\int\left[-\frac{\dot\rho}{c}\frac{\hat{\imath}}{\imath} - \rho\frac{\hat{\imath}}{\imath^2}\right]d\tau' \quad --(8)$$

Taking the divergence

$$\nabla^2 V = \frac{1}{4\pi\varepsilon_0}\int\left\{-\frac{1}{c}\left[\frac{\hat{\imath}}{\imath}\cdot(\nabla\dot\rho) + \dot\rho\,\nabla\cdot\left(\frac{\hat{\imath}}{\imath}\right)\right]\right.$$

$$\left. - \left[\frac{\hat{\imath}}{\imath^2}\cdot(\nabla\rho) + \rho\,\nabla\cdot\left(\frac{\hat{\imath}}{\imath^2}\right)\right]\right\}d\tau'$$

But $\nabla\dot\rho = -\frac{1}{c}\ddot\rho\,\nabla\imath = -\frac{1}{c}\ddot\rho\,\hat{\imath}$ as in eqn (7)

and $\nabla\cdot\left(\frac{\hat{\imath}}{\imath}\right) = \frac{1}{\imath^2}$

whereas $\nabla\cdot\left(\frac{\hat{\imath}}{\imath^2}\right) = 4\pi\delta^3(\imath)$

So

$$\nabla^2 V = \frac{1}{4\pi\varepsilon_0}\int\left[\frac{1}{c^2}\frac{\ddot\rho}{\imath} - 4\pi\rho\,\delta^3(\imath)\right]d\tau'$$

$$= \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} - \frac{1}{\varepsilon_0}\rho(r,t)$$

Confirming that the retarded potential (5) satisfies the inhomogeneous eqn (2)