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5142

M.Sc. (Final) EXAMINATION, 2016

MATHEMATICS

Paper II

Discrete Mathematics (Mathematical Methods)

Time allowed : Three Hours

Maximum Marks : 100

Part A (खण्ड 'अ') [Marks : 20]

Answer all questions (50 words each).

***All** questions carry equal marks.*

सभी प्रश्न अनिवार्य हैं । प्रत्येक प्रश्न का उत्तर **पचास** शब्दों से अधिक न हो । **सभी** प्रश्नों के अंक समान हैं ।

Part B (खण्ड 'ब') [Marks : 50]

*Answer five questions in all (250 words each), selecting **one** question from each Unit.*

***All** questions carry equal marks.*

प्रत्येक इकाई से **एक** प्रश्न चुनते हुए, कुल **पाँच** प्रश्न कीजिए । प्रत्येक प्रश्न का उत्तर *250* शब्दों से अधिक न हो । **सभी** प्रश्नों के अंक समान हैं ।

P.T.O.

Part C (खण्ड 'स') [Marks : **30**]

*Answer any two questions (**300** words each).*

***All** questions carry equal marks.*

कोई **दो** प्रश्न कीजिए । प्रत्येक प्रश्न का उत्तर **300** शब्दों से अधिक न हो । **सभी** प्रश्नों के अंक समान हैं ।

Part A

1. (*i*) Define quantifiers.

(*ii*) Define direct product of semigroups.

(*iii*) Define lattice.

(*iv*) Define Boolean algebra with example.

(*v*) Define bipartite graph.

(*vi*) Define tree.

(*vii*) Define finite state machine.

(*viii*) Define finite state automation.

(*ix*) State pumping lemma.

(*x*) Define phrase structure grammar.

Part B

Unit I

2. (a) Translate the following sentence in symbolic notation :

"Meena is inside playing the Sitar, not running outside in the rain."

(b) State the inverse, converse and contra-positive of the following :

If triangle ABC is a right-angled triangle, then :

$$|AB|^2 + |BC|^2 = |AC|^2$$

3. Let $f : S \to T$ be a homomorphism of the semigroup $(S, *)$ onto the semigroup $(T, *')$. Let R be the relation on S defined by a R b if and only if $f(a)$ and $f(b)$ for

a and b in S. Then prove :

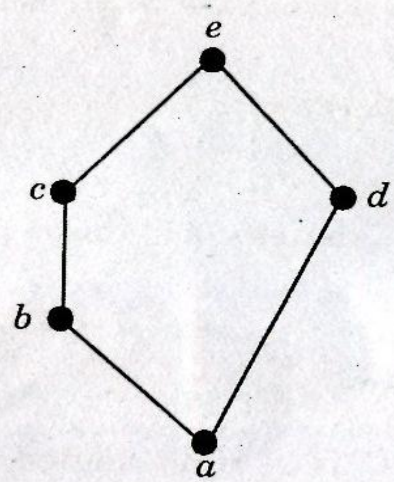
(a) R is a congruence relation

(b) (T, $*'$) and the quotient semigroup (S/R, $*$) are isomorphic.

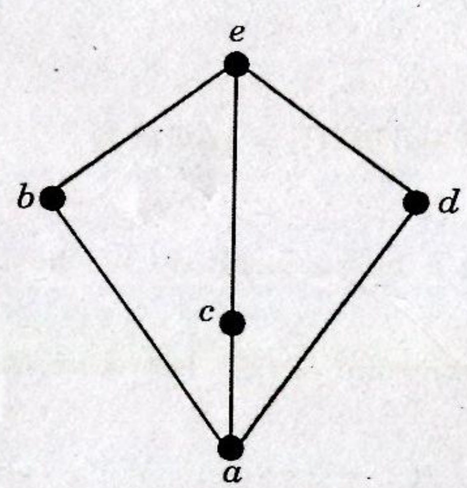
Unit II

4. State which of the following are lattices and which are not with reasons :

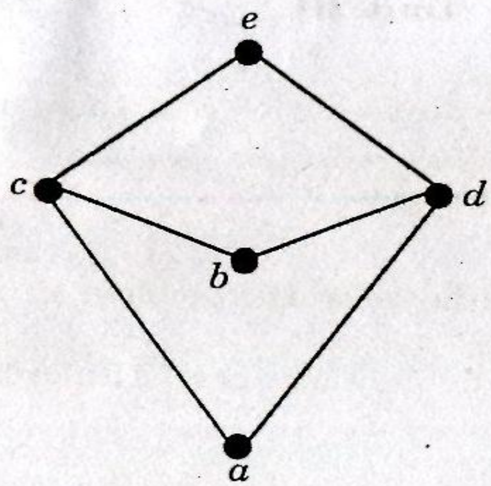
(a)



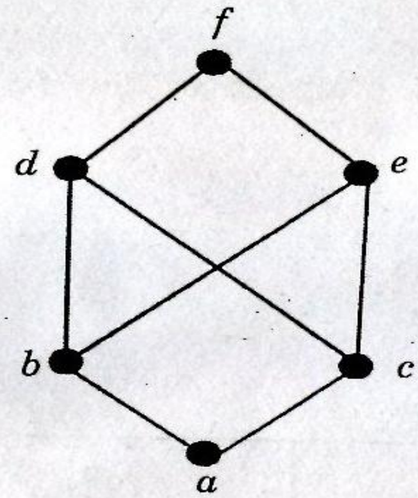
(b)



(c)



(d)



5. Prove that in a Boolean Algebra, the binary operations + and . are associative i.e., for all the elements a, b, $c \in B$:

(a) $a + (b + c) = (a + b) + c$

(b) $a(bc) = (ab)c$

Unit III

6. Prove that in every non-trivial tree there is at least one vertex of degree one.

7. Discuss the Konigsberg Bridge problem in relation to Euler graph. Also, define Euler graph, Euler path, Euler trail.

Unit IV

8. Draw the transition diagram of the finite state machine

$$M = (I, O, S, f, g, s_0)$$

given in table :

I \ S	f			g		
	a	b	c	a	b	c
s_0	s_0	s_1	s_2	0	1	0
s_1	s_1	s_1	s_0	1	0	1
s_2	s_2	s_1	s_0	0	0	1

9. Write a note on Acceptors.

Unit V

10. Let $G = (v, \Sigma, P, S)$

be the grammar where

$$V = \{S, A\}, \Sigma = \{a, b\},$$

S is the start symbol and

$$P = \{S \rightarrow bS/A \rightarrow bA/b\}.$$

Give the language generated by this grammar.

11. Define context-sensitive, context-free regular grammar with examples.

Part C

12. Let alphabet $A = \{0, 1\}$ and consider the free semigroup $(A^*, \bullet)$ generated by A. Define the following relation on A α R b if and only if α & β have the same no. of 1's. Show that R is a congruence relation on $(A^*, \bullet)$.

13. Let (A, $\leq$) be a lattice with universal lower and upper bounds 0 and 1 then for any a in A $a \vee 1 = 1$, $a \wedge 1 = a$ and $a \vee 0 = a$, $a \wedge 0 = a$.

14. Prove the Euler's formula for connected planar graph.

15. Write notes on :

(*a*) Equivalent machines

(*b*) Non-deterministic finite automata.

16. Prove that the language

$$L = \{0^k \; 1^k, \; k \geq 0\}$$

is not regular using Pumping Lemma.