5151

M. Sc. (Final) Examination, 2016

MATHEMATICS

Paper–XI

(Mathematical Theory of Statistics)

Time : Three Hours

Maximum Marks : 100

PART - A (खण्ड-अ) [Marks : 20

Answer all questions (50 words each).

All questions carry equal marks.

सभी प्रश्न अनिवार्य हैं। प्रत्येक प्रश्न का उत्तर पचास शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART - B (खण्ड-ब) [Marks : 50

Answer *five* questions (250 words each).

Selecting *one* from each unit. All questions carry equal marks.

प्रत्येक इकाई से **एक-एक** प्रश्न चुनते हुए, कुल **पाँच** प्रश्न कीजिए।

प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART - C (खण्ड-स) [Marks : 30

Answer any *two* questions (300 words each).

All questions carry equal marks.

कोई **दो** प्रश्न कीजिए। प्रत्येक प्रश्न का उत्तर 300 शब्दों से अधिक न हो।

सभी प्रश्नों के अंक समान हैं।

PART - A

UNIT - I

1. (a) What is mean by mutually exclusive events ?

(b) Give axiomatic definition of probability.

UNIT - II

(c) State the name of the distribution for whicm mean is less than variance.

(d) Write the additive property of Cauchy distribution.

UNIT - III

(e) State Chebychev's inequality.

(f) State the condition under which two lines of regression will concide.

UNIT - IV

(g) What is the value of variance of t-distribution with 'n' d.f. ?

(h) Write down the relationship between 't' and 'F' distribution.

UNIT - V

(i) What are the criterion of good estimators.

(j) Define power of the test.

PART - B

UNIT - I

2. (a) State and prove addition theorem of probability.

(b) Let A and B events with $P(A \cap B) = \frac{1}{4}$, Find

(i) $P\left(A/B\right)$

(ii) $P\left(B/A\right)$

(iii) $P(A \cup B)$

(iv) $P\left(\overline{A}/\overline{B}\right)$

3. (a) State and prove additive property of moment generating function.

(b) Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.

UNIT - II

4. For a rectangular distribution in (–a, a), p.d.f. is given by :

$$f(x)=\begin{cases}\dfrac{1}{2a} & ; \quad -a<x<a \\ 0 & ; \quad \text{otherwise}\end{cases}$$

Fidn the first four central moments and obtain β_1 and β_2.

5. Define beta and Gamma distributions. Find the mean and variance of beta distribution of first kind.

UNIT - III

6. State and prove Chebychev's inequality for a random variable 'X' with mean μ and variance σ^2.

7. (a) Given that $x = 4y + 5$ and $y = kx + 4$, are the regression lines of x on y and y on x respectively. Show that $0 < 4k < 1$. if $k = \frac{1}{16}$. Find the means of the two variables and coefficient of correlation between them.

(b) If the lines of regression of y on x and x on y are $a_1x_1 + b_1 + c_1 = 0, a_2x_2 + b_2y + c_2 = 0$ respectively, prove taht $a_1b_1 \le a_2b_1$.

UNIT - IV

8. (a) State and prove the additive property of χ^2 = distribution (χ^2 – chi square).

(b) Explain the term goodness of hit. How can the chi square (χ^2) test be used for testing the goodness of hit ?

9. (a) Find a relation between F-statistic and χ^2-. statistic.

(b) Random sample drawn from two countries gave the following data relating to the heights of adults males :

	Country A	Country B
Mean height (in inches)	67.42	67.25
Standard deviation	2.58	2.50
Number in samples	1000	1200

Is the difference between the standard deviations is significant ?

UNIT - V

10. State and prove Neyam and pearson lemma for obtaining a BCR while testing a simple hypothesis against simple alternative.

11. Explain the problem of "point estimation". Explain the term consistency and efficiency in the context of point estimation, with illustrations.

PART - C

UNIT - I

12. (a) Define the characteristics function of a random variable. Discuss its properties.

(b) For a distribution, the cumulants are given by :

$K_r = n\,[(r+1)]$ Find the characteristic function.

(c) Show that :

(i) $V(X_1 + X_2) = V(X_1) + V(X_2) + 2COV(X_1, X_2)$

(ii) $COV(AX_1, BX_2) = ab\ COV(X_1, X_2)$

UNIT - II

13. (a) Show that for a normal distribution with mean μ and variance σ^2, the central moments satisfy the relation $\mu_{2n} = (2n-1)\mu_{2n-2}\sigma^2$. Also prove that the mean deviation from the mean for this distribution is $\frac{4}{5}\sigma$ approximately.

(b) If X and Y are independent Gamma variates with parameters α and β respectively, then show that U \ $X+Y$ and $V = \frac{X-Y}{X+Y}$ are independently distributed random variables.

UNIT - III

14. (a) State and prove Inversion theorem of characteristic function. Find the density function corresponding to characteristics function $\exp\left(\frac{-t^2}{2}\right)$ of a random variable X.

(b) Using the principle of least square, obtain the normal equation for hitting a parabola $Y = a + bx = cx^2$ for n points $(X_i, Y_i)\,(i = 1, 2..n)$.

UNIT - IV

15. (a) What is students 't' statistic and derive its distribution.

(b) What is an F-test ? What are the conditions under which this test is valid ? use these conditions to obtain a test for testing the equality of variances of two normal populations, their means are knwon.

UNIT - V

16. (a) Obtain maximum likelihood estimator of θ on the basis of a simple random sample from the density $f(x; \theta) = \theta x^{\theta-1}; 0 < x < 1; \theta > 0$ and show that the estimator is always positive.

(b) Explain the following terms :

(i) Null and alternative hypothesis

(ii) Simple and composite hypothesis

(iii) Two kinds of error

(iv) Best critical region in Neyaman sense

(v) Sufficient statistic