First Year Examination of the Three Year<br>Degree Course, 2001<br>(Common for the Faculties of Arts \& Science) STATISTICS<br>Paper II<br>(Elements of Probability)<br>Time - Three Hours<br>Maximum Marks - 50<br>Attempt Five question in all, selecting ONE question from each unit.<br>All questions carry equal marks.

## SECTION A

1. (a) Define probability and explain the importance of this concept in Statistics.
(b) What is the probability of having a knave and a queen when two cards are drawn from a pack of 52 ?
2. (a) State and prove the theorem of multiplication of probability.
(b) A card is drawn from a well shuffled pack of playing cards. What is the probability that it is either a spade or an ace?
3. (a) State and prove Baye's theorem.
(b) In a bolt factory machines. A, B and C manufacture respectively $25 \%, 35 \%$ and $40 \%$ of the total. Of their output 5,4 and 2 percents are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine A ?

## SECTION B

4. (a) Explain random variable, discrete random variable, continuous random variable and their distribution functions.
(b) From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at rndom. Let the random variable $X$ denote the number of defective items in the sample. Answer the following :-
(i) Find the probability distribution of $X$.
(ii) Find $P(X<1), P(X<1)$ and $P(0<x<2)$.
5. In a continuous distribution whose relative frequency density is given by :
$F(x)=y_{0} x(2-x) ; 0<x<2$.
Find mean, variance, $\beta_{1}$ and $\beta_{2}$ and hence, show that the distribution is symmetrical.
6. What do you understand by marginal and conditional distribution? For the following bivariate probability distribution of $X$ and $Y$ find :
(i) $P(X \leq 1, Y=2)$
(ii) $P(X<1)$
(iii) $\quad \mathrm{P}(\mathrm{Y}=3)$
(iv) $\mathrm{P}(\mathrm{Y}<3)$ and
(v) $P(X<3, Y<4)$.

| Y | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | $1 / 32$ | $2 / 32$ | $2 / 32$ | $3 / 32$ |
| 1 | $1 / 16$ | $1 / 16$ | $1 / 8$ | $1 / 8$ | $1 / 8$ | $1 / 8$ |
| 2 | $1 / 32$ | $1 / 32$ | $1 / 64$ | $1 / 64$ | 0 | $2 / 62$ |

## SECTION C

7. (a) What is Sheppard's Correction? When is it used? Find the corrected moments of the following values if the magnitude of the class interval is
$\mu_{2}=43.35, \mu_{3}=9.77$ and $\mu_{4}=5508.56$
(b) Define Covariance. Show that covariance of two independent random variable is always zero. Is the converse true? Give an example to support your answer.
8. (a) Define mathematical expectation of a random variable. Show that the expectation of product of two independent variable is the product of their expectation. Is the condition of independence necessary? If not, what is the necessary condition?
(b) X and Y are independent variables with means 10 and 20 and variances 2 and 3 respectively. Find the variance of $3 X+4 Y$.
9. (a) Define moment generating function of a distribution. Explain how it helps to find moments of the distribution.
(b) Obtain the moment generating function of the following :

$$
P(x)=1 / 2 x ; x=1,2,3 \ldots \ldots
$$

Hence find mean and variance.
10. (a) Define Cumulant Generating function. State and prove additive property of cumulants.
(b) Define the characteristic function of a random variable. Write down its properties.

