

- Q.1. What is reciprocal lattice? Show that the bcc lattice is the reciprocal lattice of the fcc lattice.
- Q.2. Discuss the properties of reciprocal lattice. Give Ewald's construction for the diffraction of X-rays in terms of the reciprocal lattice.
- Q.3. What is atomic scattering factor? Derive an expression for it. Explain its significance.
- Q.4. Define geometrical structure factor. How is it related to atomic scattering factor? Derive an expression for the scattering amplitude in terms of geometrical structure factor.  
 Calculate the geometrical structure factor for simple cubic (sc), body centred cubic (bcc) and face centred cubic (fcc) lattice.
- Q.5. Show that the reciprocal to the reciprocal lattice is the direct lattice.
- Q.6. Derive Bragg's Condition in terms of reciprocal lattice vector. And interpret Bragg's law geometrically.
- Q.7. How are Brillouin zones constructed? Describe and sketch the first Brillouin zones for bcc and fcc lattices. Mention their importance in crystal analysis.
- Q.8. Explain the concept of miller indices. Deduce formula for the distance between two adjacent planes of a simple cubic lattice.
- Q.9. Show that atomic packing factor for fcc and hcp structures are same.
- Q.10. Describe a hexagonal closed-packed structure (hcp). Calculate its atomic packing fraction. Show that  $\frac{c}{a}$  ratio for an ideal (hcp) structure is  $\frac{\sqrt{8}}{3} = 1.633$ .
- Q.11. Prove that the packing fraction for a simple cubic (sc) structure is 0.52; for a body centred cubic (bcc) structure is 0.68 and for a face centred cubic (fcc) structure is 0.74.

Q.12. What is crystal structure? State the relation between crystal structure, lattice and basis. (2)

Q.13. Explain the following terms:

- (a) Crystalline solid
- (b) Amorphous solid
- (c) Lattice
- (d) Unit cell
- (e) Translation vectors

Q.14. Explain the symmetry in a crystalline solids. Describe the different symmetry elements.

Q.15. What are Bravais lattices. Discuss Bravais lattices in two dimensions. Write down symmetry operations in which each is invariant.

Q.16. Explain the crystal structure of sodium chloride (NaCl). Draw a sketch of sodium chloride lattice and write down the coordinates of the atoms in the cell.

Q.17. Explain the diamond crystal structure and give number of nearest neighbours, number of atoms per unit cell and packing fraction. Show that it has comparatively loose packing.

Q.18. Prove that for the orthorhombic system, the possible lattice plane spacing is given by

$$d_{hkl} = \left[ \left( \frac{h}{a_1} \right)^2 + \left( \frac{k}{a_2} \right)^2 + \left( \frac{l}{a_3} \right)^2 \right]^{-\frac{1}{2}}$$

If  $a_1 = 4.50 \text{ \AA}$ ,  $a_2 = 3.50 \text{ \AA}$  and  $a_3 = 2.50 \text{ \AA}$ , then calculate the lattice spacing for (110) planes.

Q.19. Find the Bragg angles and the indices of diffraction, for three lowest angle lines ~~not~~ on the powder photographs of fcc crystal:  $a = 6.0 \text{ \AA}$  and  $d = 1.54 \text{ \AA}$ .

Q.20. Prove that the volume of the unit cell of the reciprocal lattice is inversely proportional to that of the corresponding direct lattice.

Q.21. Show that, for the closest packing of spheres, the densities of the face centred cubic, body centred cubic and simple cubic lattices are approximately in the ratio 1.4:1.3:1.0.

Q.22. The primitive translation vectors of the hexagonal space lattice (3)

may be taken as

$$\vec{q}_1 = \frac{a}{2} (\hat{i} + \sqrt{3}\hat{j}), \quad \vec{q}_2 = \frac{a}{2} (-\hat{i} + \sqrt{3}\hat{j}), \quad \vec{q}_3 = c\hat{k}$$

Show that the lattice is its own reciprocal, but with a rotation of axis.  
Also prove that the volume of the unit cell in reciprocal lattice is  $\sqrt{3}a^2c/2$ .

Q.23. The diamond structure is formed by the combination of two interpenetrating fcc sub-lattice, the basis being  $(000), (\frac{1}{4}\frac{1}{4}\frac{1}{4})$ . Find structure factor of the basis, and prove that if all indices are even, the structure factor of the basis vanishes unless  $h+k+l=4n$ , where  $n$  is an integer.

Q.24. What is the angle between reciprocal lattice vectors  $\vec{G}_{100}$  and  $\vec{G}_{111}$  corresponding to a simple cubic lattice. To what plane of the direct lattice is  $\vec{G}_{100} \times \vec{G}_{111}$  perpendicular?

Q.25. Consider a lattice for which the primitive cell vectors  $\vec{q}_1, \vec{q}_2, \vec{q}_3$  have different lengths and for which the cell angles are not  $90^\circ$ . Consider two successive  $(hkl)$  planes which are a distance  $d$  apart. If  $\hat{n}$  is a unit vector in the direction perpendicular to these planes, show that

$$d = \frac{\vec{q}_1 \cdot \hat{n}}{h} = \frac{\vec{q}_2 \cdot \hat{n}}{k} = \frac{\vec{q}_3 \cdot \hat{n}}{l}$$

Hence prove that for a cubic lattice,

$$d = (h^2 + k^2 + l^2)^{-\frac{1}{2}} a$$

Q.26. For the hydrogen atom in its ground state, the number density is

$$n(r) = (\pi a_0^3)^{-1} \exp\left(-\frac{2r}{a_0}\right), \text{ where } a_0 \text{ is the Bohr radius. Show that the}$$

form factor is  $f_q = \frac{16}{(4+q^2a_0^2)^2}$ . Symbols are having their usual meanings.

Q.27. Show that volume of the first Brillouin zone is  $(2\pi)^3/V_c$ , where  $V_c$  is volume of a crystal primitive cell.

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UNIT II

- Q.28. Prove that in a one dimensional diatomic lattice, both acoustic and optical branches in dispersion curve meet the zone boundary normally.
- Q.29. Prove that the gradient of the optical branch of the dispersion curve at the maximum frequency is zero.
- Q.30. Prove that in a one-dimensional diatomic lattice for the acoustic branch
- $$\lim_{k \rightarrow 0} \omega_k \rightarrow k a \sqrt{\frac{g}{2(M_1+M_2)}}.$$
- Q.31. Prove that in a one-dimensional diatomic lattice, the two kind of atoms oscillate with amplitudes related to each other by
- $$A_2 = A_1 \left( 1 - \frac{M_1 \omega^2}{2g} \right) \sec \left( \frac{ka}{2} \right)$$
- Q.32. Show that, for the diatomic linear chain, the density of modes per unit frequency interval diverges at the maximum frequency and the frequencies on either side of the gap, while it tends to be a constant as  $\omega \rightarrow 0$ .
- Q.33. In a linear chain, all the atoms have the same mass  $M$  but are connected alternately by springs of force constants  $g_1$  and  $g_2$ . Show that the frequency-wavelength spectrum is
- $$\omega^2(q) = \left( \frac{g_1+g_2}{m} \right) \pm \frac{1}{m} \left[ (g_1+g_2)^2 - 4g_1 g_2 \sin^2 q a \right]^{1/2}$$
- Q.34. In problem 33, if  $\omega_1$ ,  $\omega_2$  and  $\omega_3$  are respectively the maximum frequency of acoustic branch, and minimum and maximum frequencies of optical branch, prove (with  $g_1 > g_2$ ) that
- $$\omega_1 = \sqrt{\frac{2g_2}{m}}, \quad \omega_2 = \sqrt{\frac{2g_1}{m}} \quad \text{and} \quad \omega_3 = \sqrt{\frac{2}{m}(g_1+g_2)},$$
- Q.35. Prove that for a one-dimensional monatomic lattice, the differential equations representing the longitudinal and transverse waves are identical.
- Q.36. If in a one-dimensional lattice,  $\alpha = \frac{m_2}{m_1} \ll 1$ , prove that the squares of the widths of the optical and acoustic branches are in the ratio  $\alpha : 4$ .
- Q.37. What is phonon? Explain the elastic scattering of photons by long wavelength phonon.

(P)

Q.38) Consider a one-dimensional crystal consisting of  $N+2$  identical atoms each of mass  $m$ , the end atoms (labelled 0 and  $N+1$ ) being fixed. Using the boundary conditions  $x_0 = x_{N+1} = 0$  for all values of  $s$ , prove that the allowed values of frequencies are given by

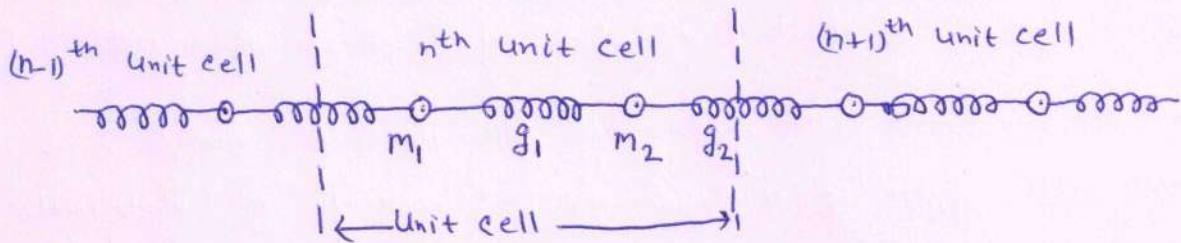
$$\omega_c = 2 \sqrt{\frac{g}{m}} \sin \left[ \frac{s\pi}{2(N+1)} \right] ; s = 1, 2, \dots, N.$$

Q.39. A more accurate description of lattice vibration is obtained if interactions beyond the nearest neighbours are included. Prove that the inclusion of  $N$ th neighbours modifies the dispersion relation of a one-dimensional monatomic lattice to the form

$$M\omega^2 = 2 \sum_{s=1}^N g_s (1 - \cos sk_a)$$

where  $g_s$  is the force constant corresponding to the  $s$ th neighbour interaction.

Q.40. In a one-dimensional diatomic lattice shown as:



It is assumed that both the masses ( $m_1$  and  $m_2$ ) of the atoms and the force constants ( $g_1$  and  $g_2$ ) of the spring differ from one another. Show that frequency spectrum is given by

$$\omega^2 = \frac{(g_1 + g_2)}{2m_1 m_2} (m_1 + m_2) \left[ 1 \pm \left\{ 1 - \frac{8g_1 g_2 m_1 m_2}{(g_1 g_2)^2 (m_1 + m_2)^2} (1 - \cos k_a) \right\}^{1/2} \right]$$

Q.41 Consider a longitudinal wave

$$u_s = 4 \cos (\omega t - sk_a)$$

which propagates in a monoatomic linear lattice of atoms of mass 'm', spacing 'a', and nearest-neighbour interaction C.

(a) Show that the total energy of wave is

$$E = \frac{1}{2} M \sum_s \left( \frac{du_s}{dt} \right)^2 + \frac{1}{2} C \sum_s (u_s - u_{s+1})^2 \quad \text{where } s \text{ runs over all atoms.}$$

(b) By substitution of  $u_s$  in this expression, show that the time-average total energy per atom is

$$\frac{1}{4} m \omega^2 u^2 + \frac{1}{2} C (1 - \cos k_a) u^2 = \frac{1}{2} m \omega^2 u^2.$$

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Q.42. Calculate the Einstein temperature of Cesium assuming that its atoms vibrate with Ionic plasma frequency.

Q.43. In an assembly of  $10^{23}$  simple harmonic oscillators, each has a frequency of  $10^{13} \text{ Hz}$ . Calculate (ignoring the zero-point energy), the mean thermal energy of the system at 20K.

Q.44. Prove, ignoring the difference between transverse and longitudinal sound, that the Debye temperature is given by

$$\theta_D = \frac{\pi c}{k_B} \left( \frac{6\pi^2 N}{V} \right)^{1/3} \quad \text{where } c \text{ is the sound velocity.}$$

Q.45. Show that, because of the changes induced by anharmonicity in a solid, the specific heat at high temperatures might be expected to vary not as

$$C_V = 3Nk_B, \text{ but as}$$

$$C_V = 3Nk_B (1 + a k_B T), \text{ where } a \text{ is proportional to } V^2.$$

Q.46. The thermal conductivity of an artificial sapphire crystal 3mm in diameter reaches a sharp maximum at 30K. Estimate the maximum value of thermal conductivity if for sapphire,  $\theta_D = 1600\text{K}$ , speed of sound =  $10^4 \text{ m s}^{-1}$ , and for  $T \ll \theta_D$ ,  $C_V = 0.1 T^3 \text{ J m}^{-3} \text{ K}^{-1}$ .

Q.47. On the basis of result of the problem 46, estimate the thermal conductivity of sapphire at liquid nitrogen temperature (80K).

Q.48. Prove that the zero-point energy of a solid on the Debye model is given by  $\frac{9}{8} \pi R \theta_D$ .

Q.49. Prove that for a two-dimensional Debye solid, the specific heat at low temperature varies as  $T^2$ .

Q.50. At very low temperatures, the specific heat of rock salt varies with temperature according to Debye  $T^3$ -law

$$C_V = K \frac{T^3}{\theta_D^3}$$

where  $K = 464 \text{ cal mol}^{-1} (\text{K})^{-1}$  and  $\theta_D = 281\text{K}$ . How much heat is required to raise the temperature of 2 moles of rock salt from 10 to 50K?

Q.51. In an insulator, the phonon mean free path and sound velocity are respectively  $3 \times 10^{-6}$  cm and  $10^5$  cm/s. The corresponding parameters for a monovalent metal are  $10^{-5}$  cm and  $10^8$  cm/s (=Fermi velocity). If  $C_V = 0.1 R$  for the metal, estimate the ratio of thermal conductivities of the metal and the insulator. (7)

Q.52. Neglecting the thermal variation of specific heat, prove that the temperature dependence of thermal expansion is given by

$$\frac{d\alpha}{dT} = -\frac{9gfc_V^2}{8c^4r_0}$$

where  $r_0$  is the equilibrium interatomic distance.

Q.53. Estimate the Debye temperature of gold, if its atomic weight is 197, the density is  $1.9 \times 10^4$  kg m $^{-3}$  and the velocity of sound is  $2100$  ms $^{-1}$ .

Q.54. Prove that at low temperatures, the temperature dependent part of the internal energy is

$$U = \frac{\pi^2 V}{10\hbar^3 \bar{c}^2} (k_B T)^4$$

where  $\bar{c}$  stands for the mean velocity of sound obtained by averaging the inverse cube of the actual phase velocity over branches as well as angles.

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Q.55. Show that the fraction ( $\alpha$ ) of electrons within  $k_B T$  of the

Fermi level is given by

$$\alpha = \frac{3}{2} \frac{k_B T}{E_F}, \text{ calculate it for Sodium (Na).}$$

Q.56. Prove that the average kinetic energy per electron in a metal is given by

$$E_{av} = \frac{3}{5} E_F$$

Q.57. The metal bonnevillium, which has a single electron in an s-state, solidifies in a close-packed monolayer structure. Show that, for  $N$  atoms per square meter

$$E_F = \frac{\pi^2 h^2 N}{4 \pi m}$$

Q.58. Derive an expression for the Fermi energy of a Free-electron metal at zero temperature. Using the data given and any other constants, evaluate the

Fermi of the alkali metals:

	Li	Na	K	Rb
Density (g/cm³)	0.534	0.971	0.86	1.53
Atomic weight	6.939	22.99	39.202	85.47

How would you measure the Fermi energy experimentally for these metals?

Q.59. The number of free electrons per cubic metre of sodium is  $2.5 \times 10^{28}$ . Calculate the Fermi energy and Fermi velocity.

Q.60. Calculate the Fermi energy of silver from the data given below:

Density of silver =  $10.5 \text{ gm/cm}^3$ ; atomic weight =  $108$ ;  $h = 6.62 \times 10^{-34} \text{ Joule-sec}$ ;  $m = 9.1 \times 10^{-31} \text{ kg}$ , Avogadro's number =  $6.02 \times 10^{23} \text{ atoms/gm-atom}$ .

Q.61. Show that Fermi energy at room temperature is given by

$$E_F = E_{F_0} \left[ 1 - \frac{k^2}{12} \left( \frac{kT}{E_{F_0}} \right)^2 \right] \quad \text{where } E_{F_0} \text{ is the Fermi energy at } 0^\circ\text{K.}$$

Q.62. What is the cause of failure of free electron theory?

Q.63. Use the equation  $m \left( \frac{dv}{dt} + \frac{v}{T} \right) = -eE$  for the electron drift velocity  $v$ .

Show that the conductivity at frequency  $\omega$  is

$$\sigma(\omega) = \sigma(0) \left( \frac{1 + i\omega\tau}{1 + (\omega\tau)^2} \right) \quad \text{where } \sigma(0) = \frac{ne^2\tau}{m}.$$

Q.64. The atom  $\text{He}^3$  has spin  $\frac{1}{2}$  and is a fermion. The density of liquid  $\text{He}^3$  is  $0.081 \text{ g cm}^{-3}$  near absolute zero. Calculate the Fermi energy  $E_F$  and the Fermi temperature  $T_F$ .

Q. 65. Using density of states at the Fermi level, prove that the electronic specific heat is given by

$$C_e = \left( \frac{\pi^2 N k_B^2}{2 E_F} \right) T$$

Q. 66. Calculate the electronic specific heat of gm mole of Cu at 300K. At what low temperature are the electronic and lattice specific heats of Cu equal to one another? Debye temperature for Cu is 348 K.

Q. 67. A Cu wire of diameter 2 mm carries 10A of current. Find the drift velocity of conduction electrons.

Q. 68. If the Fermi energy of Na is 3.1 eV and electrical conductivity is  $2.1 \times 10^{17} \text{ esu}$  at 0K, calculate the relaxation time.

Q. 69. Using the Drude formula, calculate the mean free path of K (Potassium), if its Fermi energy is 2.1 eV and electrical conductivity is  $1.5 \times 10^{17} \text{ esu}$  at 0K.

Q. 70. Calculate the density of conduction electrons in Na, if its lattice parameter  $a = 4.3 \text{ \AA}$ . All calculate the Hall Coefficient.

Q. 71. Sodium has a bcc structure of cell side  $4.28 \text{ \AA}$ . Estimate, on a free electron model, the e.m.f. generated when a current of 100mA passes along a sample of sodium 5mm wide and 1 mm thick in a perpendicular field of 0.1 T.

Q. 72. Calculate the Hall voltage across the width of a semiconducting specimen from the following data:

width of specimen = 0.1 m; thickness of specimen = 0.01 m, field applied perpendicular to both width and length = 0.6 T, current flowing length wise = 10mA; Hall coefficient =  $3.84 \times 10^{-4} \text{ m}^2/\text{C}$ .

Q. 73. Prove that for a two-dimensional metallic system, the density of states per unit area is given

$$\rho(E) = \frac{2\pi}{h^2} g_n m^*$$

where  $m^*$  is the effective mass for electron and  $g_n$  for simple bands is the degeneracy factor for the atomic level.

Q. 74. Calculate the density of states of  $1 \text{ m}^2$  of Na at Fermi level for  $T=0\text{K}$ , if  $m^*/m = 1.2$ .

Q. 75. A free-electron system in which each level is six-fold degenerate has its energy filled upto  $E = (2\pi^2)^{1/3}/a$ , where 'a' is the interatomic spacing. Calculate the number of conduction electrons per atom.

Q.76. Using the tight binding model, prove that the energy-wavevector dispersion relation for one-dimensional crystal of lattice constant  $a$  is

$$E(k) = E_0 - \alpha - 2\beta \cos kq$$

- (a) Find the value of  $k$  at which the velocity of an electron is maximum.  
 (b) Obtain an expression for  $m^*$  as a function of  $k$ .

Q.77. Prove that the inverse mass tensor of an electron is

$$\frac{1}{m^*} = \frac{1}{k^2} \nabla_{\mathbf{k}} \nabla_{\mathbf{k}} E.$$

Q.78. Show that near the bottom of an s-band in a simple cubic structure, the density of states on the LCAO method is

$$N(E) = \frac{1}{2\pi^2 a^3 |\beta|^{3/2}} [E - E(0)]^{1/2}$$

where  $E(0)$  is the energy at  $\mathbf{k}=0$ .

Q.79. The face-centred cubic lattice has 12 neighbours at  $\mathbf{q}(0, \pm 1, \pm 1)$ ,  $\mathbf{q}(\pm 1, 0, \pm 1)$ ,  $\mathbf{q}(\pm 1, \pm 1, 0)$ . Use the tight binding method to show that the energy band constructed from an atomic s-state, upto nearest neighbours, is given by

$$E(k) = E_0 - \alpha - 4\beta [\cos k_x q \cos k_y q + \cos k_y q \cos k_z q + \cos k_z q \cos k_x q]$$

where  $E_0$  is the free atom energy.

Q.80. The body-centred cubic lattice has eight nearest neighbours ( $\pm 1, \pm 1, \pm 1$ ). Use LCAO method to show that the energy band constructed from an atomic s-state, upto nearest neighbours, is given by

$$E(k) = E_0 - \alpha - 8\beta \cos k_x q \cos k_y q \cos k_z q$$

Q.81. Prove that the current carried by Bloch electrons is given by

$$j = - \left( \frac{eU(k)}{m} \right) \mathbf{k}$$

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Q.82. The valence band of a simple cubic metal has the form

$$E = AK^2 + B$$

where  $A = 10^{-38} \text{ J m}^2$ . Calculate  $m^*/m$ .

Q.83. Sodium has a density of  $0.971 \text{ g cm}^{-3}$  and an atomic weight of 22.99. Calculate its Fermi energy.

Q.84. Neutron stars consist of a Fermi gas of neutrons. The density in a typical neutron star is  $5 \times 10^{16} \text{ kg/m}^3$ . Calculate the Fermi energy and the Fermi speed of the neutrons.

Q.85. Calculate the number of electrons per atom for which the spherical Fermi surface of an fcc metal first touches the face of the zone boundary.

Q.86. Assuming that silver is a monovalent metal with a spherical Fermi surface, calculate:

- (i) Fermi energy and Fermi temperature
- (ii) Radius of the Fermi sphere
- (iii) Fermi velocity
- (iv) Cross-sectional area of the Fermi surface.

Q.87. If an electric field equal to  $1.0 \text{ V/m}$  is applied to a specimen of sodium metal, find the drift velocity of the conduction electrons and the displacement  $\Delta k$  of the Fermi surface.

Q.88. In a simple cubic quasi-free electron metal, the spherical Fermi surface just touches the first Brillouin zone. Calculate the number of conduction electrons per atom.

Q.89. Prove that the electron density at which the free electron spherical Fermi surface first touches the zone boundary in a bcc cubic metal is  $2\sqrt{2}(ka^2)/2\pi$ , where 'a' is the lattice parameter.

Q.90. Show that at low temperature, the Fermi energy varies with temperature as

$$E_F(T) = E_F(0) - \frac{(k_B T)^2}{6S(E_F)} \left( \frac{\partial S}{\partial E} \right)_{E=E_F}$$

where  $S(E)$  is the density of electronic states per unit energy range.

Q.91. A typical cross-sectional area of a Fermi surface in  $k$ -space is  $10^{20} \text{ m}^{-2}$  and the largest field used in experiments is about  $10 \text{ T}$ . Estimate the quantum number of the relevant electronic orbit.

Q.92. The de Haas van Alphen effect is studied in Cu at a field of 10T. (12)  
 what is the order of the maximum temperature which can be tolerated while still getting a good effect.

Q.93. In observing a good de Haas van Alphen effect, it is necessary that collision broadening should be comparatively small. If the impurity density  $n$  and electron collision time  $\tau$  are related by  $n\tau = 10^{14} \text{ m}^{-3}\text{s}$ , upto what density of impurities can the effect still be readily observed?

Q.94. Prove that the electrical conductivity  $\sigma$  is connected to the total free area ( $S'$ ) of the Fermi surface by

$$\sigma = \frac{e^2 S' \text{del}}{4\pi^2 k} \quad \text{where del is an average mean free path for electrical conductivity.}$$

Q.95. The Fermi energy of Al is 12eV and its electrical conductivity is  $3 \times 10^3 \Omega^{-1}\text{m}^{-1}$ . Calculate the mean free path of the conduction electrons and their mean drift velocity in a field of  $1000 \text{ Vm}^{-1}$ . For Al, atomic weight = 27, density =  $2700 \text{ kg/m}^3$ .

Q.96. Prove that the Meissner effect is consistent with the disappearance of resistivity in a superconductor.

Q.97. Show that when superconductivity is destroyed by the application of a magnetic field, the material will cool.

Q.98. A superconducting tin ( $T_{\text{Sn}}$ ) has a critical temperature of 3.7 K in zero magnetic field and a critical field of 0.0306 T at 0K. Find the critical field at 2 K.

Q.99. For a specimen of  $V_3Ge$ , the critical fields are respectively  $1.4 \times 10^5$  and  $4.2 \times 10^5$  ampere-turn/m for 14K and 13K. Calculate the transition temperature and critical fields at 0K and 4.2 K.

Q.100. Consider a sphere of a type I superconductor with critical field  $H_c$ .  
 (a) Show that in the Meissner regime the effective magnetization  $M$  within the sphere is given by  $-8\pi M/3 = B_0$ , the uniform applied magnetic field.

(b) Show that the magnetic field at the surface of the sphere in the equatorial plane is  $\frac{3B_0}{2}$ . (It follows that the applied field at which the Meissner effect starts to break down is  $2H_c/3$ ). The demagnetization field of a uniformly magnetized sphere is  $-4\pi M/3$ .

Q.101. Consider a junction of rectangular cross section with a magnetic field  $B$  applied in a plane of the junction, normal to an edge of width  $w$ . Let the thickness of the junction be  $T$ . Assume for convenience that phase difference of the two superconductors is  $\frac{\pi}{2}$  when  $B=0$ . Show that the dc current in the presence of the magnetic field is

$$J \approx J_0 \frac{\sin(\omega T B e / \hbar c)}{(\omega T B e / \hbar c)}.$$

Q.102. What is superconductivity? For dc Josephson effect, prove that the current  $J$  of superconductor pairs across the junction depends on the phase difference  $\delta$  as

$$J = J_0 \sin \delta = J_0 \sin(\theta_2 - \theta_1)$$

Q.103. In ac Josephson effect, show that the superconducting current is given by

$$J = J_0 [\delta(0) - (2eVt/\hbar)].$$

Q.104. Describe outline of the BCS theory of superconductivity. Find an expression of London equations in superconductivity and define penetration depth. Explain type-I and type-II superconductors.

Q.105. Calculate the penetration depth for a superconducting material having electron density  $n = 4 \times 10^{26} / m^3$ . What does happen to the penetration depth as the critical temperature is approached?

Q.106. Find the critical magnetic field for wire of lead at 4.2 K. A parabolic dependence of  $H_c$  upon  $T$  may be assumed.  $T_c$  and  $H_0$  for lead is 7.18 K and  $6.5 \times 10^4$  ampere/metre respectively.

Q.107. The transition temperature of mercury with an average atomic mass of 200.59 a.m.u. is 4.153 K. Determine the transition temperature of one of its isotopes  $^{80}\text{Hg}^{204}$ .

Q.108. Applying Maxwell's equations to a perfect diamagnetic substance, show that the magnetic field inside it does not change with time.

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UNIT - V

(14)

Q.109. Using the wavefunction for the ground state of Hydrogen atom

$$\psi_1 = \frac{1}{(\pi a_0^3)^{1/2}} \exp\left(-\frac{r}{a_0}\right)$$

Calculate the diamagnetic susceptibility of a mole of atomic Hydrogen, where  $a_0$  is the Bohr radius.

Q.110. Estimate the order of magnitude of the diamagnetic susceptibility of Copper, if its atomic radius is  $1\text{\AA}$ . Assume that only one electron per atom contributes and the lattice constant is  $3.608\text{\AA}$ .

Q.111. The magnetic moment of an electron in the ground state of the Hydrogen atom is 1 Bohr magneton. Calculate the induced magnetic moment in a field of 1 weber/m<sup>2</sup>.

Q.112. Suppose an atom with a spherically symmetric charge distribution  $\rho(r)$  is placed in a magnetic field  $H$ . Show that the induced diamagnetic current produces a field at the nucleus given by

$$\Delta H = -\left(\frac{eH}{3mc^2}\right) \Phi_E(0)$$

where  $\Phi_E(0) = \int_0^\infty \frac{\rho(r)}{r} 4\pi r^2 dr$  is the electrostatic potential at the nucleus.

Q.113. For a paramagnetic gas of  $N$  atoms/cm<sup>3</sup> and  $L=0, S=\frac{1}{2}$ , calculating the number of atoms in the two levels at temperature  $T$  and in the field  $H$ , prove that the resultant magnetisation is

$$M = \frac{1}{2} N g \mu_B \tanh\left(\frac{g \mu_B H}{2k_B T}\right)$$

Q.114. There are  $10^{22}$  cm<sup>-3</sup> atoms in a paramagnetic gas, defined by  $L=0, S=\frac{1}{2}$ , if the temperature of the gas is 4K, find the number of atoms in the two levels produced in the magnetic field of 25000 oersted.

Q.115. A system of electron spins is placed in a magnetic field of 2Wb/m<sup>2</sup> at a temperature  $T$ . The number of spins parallel to the magnetic field is twice as large as the number of antiparallel spins. Determine  $T$ .

Q.116. A magnetic field is applied to a salt containing  $\text{Cu}^{2+}$  ions, which have nine electrons in the 3d-shell. What magnetic field must be applied to a salt containing  $\text{Cu}^{2+}$  when at 1K, so that 99% of the ions are in the lowest energy state?

Q.117. A paramagnetic system of electric spin magnetic dipole moment is placed in an applied field of  $10^5 \text{ A/m}$ . Calculate the average magnetic moment per dipole at 300K and at 0.3K. Also calculate the fractional number of spins which are parallel and antiparallel to the field.

Q.118. (a) Find the magnetization as a function of magnetic field and temperature for a system of spins with  $S=1$ , moment  $M$ , and concentration  $n$ . (15)

(b) Show that in the limit  $4B \ll kT$  the result is  $M \approx (2n\mu^2/3kT)B$ .

Q.119. Derive the magnon dispersion relation for a spin  $S$  on a simple cubic lattice.

Q.120. Calculate the internal field for ferromagnetic iron which has a Curie temperature of 1043K and an effective moment of 2.2 Bohr magneton per ion.

Q.121. The ferromagnetic europium oxide has a Curie temperature of 70K. The europium ion has  $J=\frac{7}{2}$  and  $g=2$ . Assuming the internal field model, determine the ratio of magnetization at 300K in a field of 0.01 tesla to that at 0K.

Q.122. The Curie temperature of ~~iron~~ iron is 1043 K. Assume that iron atoms, when in the metallic form have moments of 2 Bohr magnetons per atom. Iron is a body centred cubic with the lattice parameter  $a = 2.86\text{\AA}$ . Calculate: (a) the saturation magnetisation; (b) the Curie constant; (c) the Weiss field constant; (d) the magnitude of the internal field.

Q.123. Show that the exchange integral is given by

$$J = \frac{3k_B T_c}{2zS(S+1)}$$

where  $S$  is the total spin and  $z$  is the number of nearest neighbours; prove also that the exchange energy per atom is  $-2JS^2z$ .

Evaluate  $J$  for bcc iron with  $T_c = 1093\text{K}$ , and assume  $S=1$ .

Q.124. Derive the formula

$$d = \frac{3k_B T_c}{Ng^2 S(S+1) \mu_B^2}$$

for the Weiss molecular field constant  $d$ .

Q.125. Prove that paramagnetic susceptibility of conduction electrons in a metal is given on the free electron model by

$$\chi = \frac{3N\mu^2}{2E_F} \quad \text{where } k_B T \ll E_F \quad (\text{Fermi energy}).$$

Q.126. Given that the Curie Temperature of a Ferromagnet is  $727^\circ\text{C}$ , what is the order of the order of the magnitude of the exchange integral? From this estimate the internal field.

Q.127. A typical magnetic field achievable with an electromagnet with iron core is about ~~one~~ 1 Tesla =  $10^4\text{G}$ . Compare the magnetic interaction energy  $\mu H$  of an electron spin magnetic dipole moment with  $k_B T$  at room temperature and show that at ordinary temperature the approximation  $(k_B T/\mu H) \gg 1$  is valid.

(16)

Q.128. Consider He atom in its ground state. The mean radius in the Langevin formula may be approximated by the Bohr radius

$\bar{r} = \frac{k^2}{me^2} = 0.53 \times 10^{-8} \text{ m}$ , Using  $N = 27 \times 10^{23}$  per  $\text{cm}^3$  for the atomic density of Helium gas and  $\frac{e^2}{mc^2} = 2.8 \times 10^{-13} \text{ cm}$ . Calculate the diamagnetic susceptibility of a Helium atom.

Q.129. Prove that the classical dipole  $M$  in a magnetic field  $B$  precesses around the field with a frequency equal to the Larmor frequency  $\omega_L = \frac{eB}{2m}$ . Calculate the Larmor frequency in hertz, for the orbital moment of the electron in a field  $B = 1 \text{ T}$ .

Q.130. Discuss ferromagnetism, what are ferrites? Discuss their applications. Explain different properties of ferromagnetic substances.

Q.131. Describe the classical theory of diamagnetism. Obtain an expression for the diamagnetic susceptibility of material.

Q.132. Explain domain theory of ferromagnetism. Discuss the origin of domains.

Q.133. Describe the quantum theory of paramagnetism and explain how it removes the shortcomings of the Langevin's theory.

Q.134. Explain paramagnetism, diamagnetism and ferromagnetism. What is Bohr magneton?

Q.135. Explain Heisenberg's theory of ferromagnetism and set up the relation between exchange integral and exchange field constant.